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A model formulation for the prediction of churning power loss in worm gear transmission

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Abstract

Churning power loss is a complex process that causes significant loss of energy under splash lubrication of gear units. A set of empirical equations are available that allows for precise churning power losses calculation for only the parallel axis gears. The main objective of this study is to formulate a new mathematical model for worm gear based on the experiment result. The mathematical model is formulated using the dimensional analysis method and has been experimentally validated over a wide range of speeds in terms of revolutions, types of lubricants, lubricant temperature, and immersion depths. The simple direct torque measurement method was used to measure the churning loss. The influence of inertia force, viscous force, and gravitational forces were also investigated therefore correlations with Froude and Reynolds numbers are presented. Hardened steel and bronze were used as a material combination of worm and worm gear, respectively. A new formula for worm gear under splash lubrication has been derived and validated by comparison with experimental evidence. A very little discrepancy was found between the experiment result and the mathematical equation.

1. Introduction

Worm gears are used in a variety of applications, including industrial, heavy equipment, and even consumer products. Despite their low efficiency, they may produce extremely high reduction ratios and are in many situations self-locking [1]. The power losses in gear are split into two categories: (i) load-dependent (mechanical) and (ii) load-independent (spin) power losses due to windage losses, oil churning losses, and gear and bearing oil sealing losses. Churning power losses occur when gears drag in splash oil while windage power losses occur when gears drag in the air [2-4].

This study has two main objectives. The first objective is to formulate the mathematical model based on the experimental result. The second objective is to validate the formulated churning power loss model by performing experiments with a new test matrix having a new level of each parameter. The prediction method for churning power loss of worm gear is developed by using statistical analysis of the experiment.

Mann and Marston [5] analysed the simplest example of churning power loss by experimenting on a small disc rotating in the fluid. Daily and Nece [6] experimented on enclosures and bladed discs. Terekhov [7] and Boness [8] were one of the first to investigate the churning losses in dip-lubricated geared transmissions for a wide range of operating conditions. The churning torque is expressed in terms of dimensionless drag torque C_m as Equation (1). They introduced several empirical equations based on different Reynolds and Froude numbers which are shown in Equations (2) to (4).

$$C_{ch} = \rho \omega^2 r^4 b C_m . \quad (1)$$

For laminar flows (10 < Re < 2250) if $Re^{-0.6} Fr^{-0.25} > 8.7 \times 10^{-3}$

$$C_{\mathrm{m}} = 4.57 Re^{-0.6} Fr^{-0.25} \left(\frac{h}{r}\right)^{1.5} \left(\frac{b}{r}\right)^{-0.4} \left(\frac{V}{V_{\mathrm{o}}}\right)^{-0.5} \quad (2)$$

otherwise

$$C_{\mathrm{m}} = 2.63 Re^{-0.6} Fr^{-0.25} \left(\frac{h}{r}\right)^{1.5} \left(\frac{b}{r}\right)^{-0.17} \left(\frac{V}{V_{\mathrm{o}}}\right)^{-0.73} \quad (3)$$

For turbulent flows (2250 < Re < 36.000)

$$C_{\mathrm{m}} = 0.373 Re^{-0.3} Fr^{-0.25} \left(\frac{h}{r}\right)^{1.5} \left(\frac{b}{r}\right)^{-0.124} \left(\frac{V}{V_{\mathrm{o}}}\right)^{-0.574} \quad (4)$$

The churning torque formula provided by Boness [8], who studied the drag torque created by discs of various geometries and gears spinning up to 3000 rpm at various immersion depths in water, SAE 15W-40 or SAE 10W oil, is represented in terms of a dimensionless torque coefficient C_{m} as shown in the following equations.

$$C_{\mathrm{ch}} = \frac{1}{2} \rho \omega^2 S_{\mathrm{m}} r^3 C_{\mathrm{m}}. \quad (5)$$

For laminar flows (10 < Re < 2000)

$$C_{\mathrm{m}} = \frac{20}{Re}. \quad (6)$$

For laminar flows (2000 < Re < 100.000)

$$C_{\mathrm{m}} = 8.6 \cdot 10^{-4} Re^{1/3}. \quad (7)$$

For turbulent flows (100.000 < Re)

$$C_{\mathrm{m}} = \frac{5 \cdot 10^8}{Re^2}, \quad (8)$$

$$\Delta P = \frac{1}{2} \rho \omega^3 S_{\mathrm{m}} R_{\mathrm{p}}^3 C_{\mathrm{m}}, \quad (9)$$

$$C_{\mathrm{m}} = 17.7 Fr^{0.68} \left(\frac{u-1}{u^8}\right) \left[1 - \left(\frac{h}{R_{\mathrm{p}}}\right)_{\mathrm{gear}}\right]. \quad (10)$$

Daily and Nece [6], Mann and Marston [5] and Luke and Olver [9] investigated the losses of a single dish or gear and presented their equation. In actual transmission, the most common situation suggests that gear and pinion are in a mesh position so Changenet and Velex [10] conducted several experiments on gear pairs with the same module to quantify the additional loss (ΔP) associated with counter-clockwise rotations. As discussed above, many authors developed an empirical model for churning power loss of parallel axis gear [7-11] and some authors developed the same for bevel gear [4,12]. In this paper, a new empirical equation has been developed based on the dimensional analysis of the experiment results for worm gear.

2. Formulation for churning power loss of worm gear

The test rig is developed for churning loss measurements as shown in Figure 1. The churning power loss was measured by the direct torque measurement technique. It consists of the electric motor, torque sensor and variable frequency drive (VFD). An electric motor drives the worm gear using a flexible coupling through a torque sensor. The foot-mounted bearings are placed on both sides to reduce the vibration of the torque sensor. The resisting torque is directly estimated by the rotational torque sensor of precision ± 0.01 Nm. For measurement and regulating the speed of the worm, a variable frequency drive (VFD) has been used. The temperature of the housing and the temperature of the lubricant were both measured using a temperature sensor. The oil level can be measured by an oil level indicator. The pressure of the air inside the gearbox was measured using a gauge mounted on the top of the gearbox. The non-return air valves were arranged at top of the gearbox to reduce or diminish the effect of windage loss. The volume of the test gearbox was kept constant (180 × 180 × 280 mm). The gearbox was filled with oil according to the test matrix.

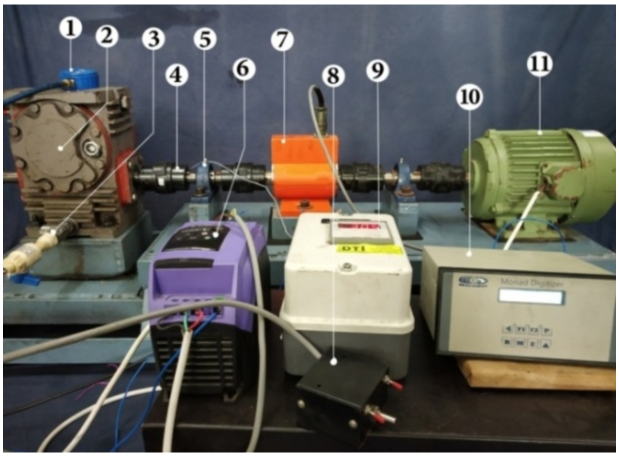


Figure 1. Test rig to investigate churning power loss of worm gear (1 – pressure gauge; 2 – worm gearbox; 3 – provision for oil level indicator; 4 – jaw type coupling; 5 – foot-mounted bearing; 6 – variable frequency drive (VFD); 7 – torque sensor; 8 – VFD regulator; 9 – temperature indicator; 10 – digital controller for torque sensor; 11 – AC motor)

To calculate the input power $P_{(\text{immersed})}$, the torque and speed were measured with the help of a torque sensor and variable frequency drive, respectively. Before conducting another set of measurements, the gearbox oil temperature was allowed to cool to room temperature. When the worm and worm wheel was not submerged in oil, the power loss $P_{(\text{non-immersed})}$ was calculated at the same speed as in the immersed situation. A little amount of oil was provided to the gear work area and the bearing area for the non-immersed condition. Assuming that the friction power loss estimates for both (immersed and non-immersed) the circumstances were about comparable [13].

$$P_{(\text{immersed})} = P_{\text{wf}} + P_{\text{wc}} + P_{\text{bf}} + P_{\text{bc}} + P_{\text{s}}, \quad (11)$$

$$P_{(\text{non-immersed})} = P_{\text{wf}} + P_{\text{bf}} + P_{\text{s}}. \quad (12)$$

The difference in power loss between the two lubrication conditions (immersed and non-Immersed) was due to the gears and bearings churning [14-15].

Table 1 shows the controllable variables for the experiment and Table 2 represents the lubricant properties at various temperatures.

Table 1. Experimental parameters with their levels

Control factor	Unit	Level 1	Level 2	Level 3
Oil temperature	°C	30	40	50
Speed of worm	rpm	1000	1200	1400
Oil volume	l	1.5	2.1	2.7

Table 2. Lubricant property for the experiment

Type of oil	Oil A	Oil B	Oil C
Name	mineral oil (EP 140)	synthetic oil (PAO 320)	mineral oil (SP 320)
Kinematic viscosity at 40 °C, mm^2/s	312	330	184
Kinematic viscosity at 100 °C, mm^2/s	33	35.5	24.1
Viscosity index	95	162	90
Density, kg/m^3	880	790	870

Based on the experiments, the parameters which are influential on the churning power loss of worm gear have been categorised in Table 3. The churning power P_c can be expressed as a function of those parameters.

$$P_c = f(R, h, V, S_m, \varnothing, \omega, \mu, g, \rho), \quad (13)$$

Table 3. Parameters for dimensional analysis

Geometric parameters	Centre distance (X)
	Radius of worm shaft (R)
	Immersion depth (h)
	Oil volume (V)
	Immersed surface area (S_m)
	Reduction ratio ($\varnothing$)
Fluid dynamic parameters	Angular velocity (ω)
	Oil dynamic viscosity (μ)
	Gravitational acceleration (g)
	Oil density (ρ)

If the drag force acts on the body by the fluid (according to the fluid drag equation)

$$F_d = \frac{1}{2}\rho v^2 S_m C_m, \quad (14)$$

$$\text{Torque} = F_d \cdot \text{radius}, \quad (15)$$

$$\text{Torque} = \frac{1}{2}\rho v^2 S_m C_d R, \quad (16)$$

$$\text{Torque} = \frac{1}{2}\rho R^3 \omega^2 S_m C_m. \quad (17)$$

According to the π theorem from the dimensional analysis, the dimensionless churning torque is described by

$$C_m = f(\rho, R, h, V, \mu, \varnothing, g). \quad (18)$$

Dimensional analysis (Buckingham π theorem) has been used to predict the new equation for dimensionless drag torque (C_m) [13,16-18]. The normalised dimensionless torque (C_m) equation is obtained from the Buckingham π theorem as follows

$$C_m = \psi \left(\frac{h}{R}\right)^{\gamma} \left(\frac{V}{R^3}\right)^{\delta} Re^{-\alpha} Fr^{-\beta} \varnothing^{-\varepsilon}, \quad (19)$$

where α, β, δ, ϵ, γ and ψ are constants and which are adjusted from the statistical analysis of the experiment result.

Reynolds number (Re) is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\omega R^2}{\gamma}. \quad (20)$$

Froude number (Fr) is defined as the ratio of the inertia force to gravitational force as shown below.

$$Fr = \frac{\omega^2 R}{g}. \quad (21)$$

For considering the radius of the rotational member for Reynolds number (*Re*) and Froude number (*Fr*), only the radius of the worm shaft is considered. The radius of the worm wheel is not considered due to its too low speed. For considering the radius for immersion depth to radius ratio (h/R) and volume to radius ratio (V/R^3), radius (R) is considered as the summation of both radiuses ($R_1 + R_2$). It becomes the centre distance (X) as shown in Figure 2. So Equation (19) is revised in the form of Equation (22).

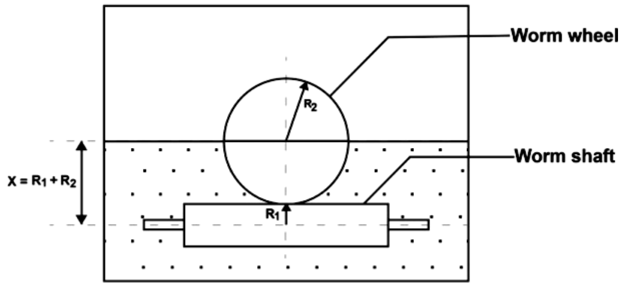


Figure 2. Centre distance of worm gear (X) for empirical equation

$$C_m = \psi\left(\frac{h}{X}\right)^{\gamma}\left(\frac{V}{X^3}\right)^{\delta} Re^{-\alpha} Fr^{-\beta} Ø^{-\varepsilon}. \tag{22}$$

2.1 Dimensionless numbers

Three types of forces can impact the worm gear's churning power losses: viscous force, inertia force, and gravity force of the lubricant. Dimensionless numbers such as Reynolds number (*Re*) and Froude number (*Fr*) might be used to represent these forces [19].

Reynolds number (*Re*). The ratio of the inertia force to the viscous force is known as the Reynolds number. To determine the exponent, the test data for the worm gear submerged in different oils with the same immersion depth were employed. All other test conditions remained the same except for the viscosity of the lubricant. Thus, the exponent can be derived from the two lubricant test results at the same rotational speed, gear pair and direction of rotation, and expressed in terms of the dimensionless churning torques and the viscosity of the oil. At high-speed, inertia force is more dominant than viscous force. However, here viscous force is more dominant than inertia force.

$$\left(\frac{C_{ma}}{C_{mb}}\right) = \left(\frac{Re_a}{Re_b}\right)^{\alpha}, \tag{23}$$

with the index "a" for oil A and index "b" for oil B, all other variables concerning gear pair, worm speed, lubricant volume and lubricant temperature did not change.

Table 4. Calculation of α at lubricant temperature 40 °C

No.	Type of oil	Speed, rpm	*V*, l	C_m	*Re*	α
1	Oil A	1000	1.5	0.0163	268.512	– 1.0308
	Oil B			0.0172	253.866	
2	Oil A	1200	1.5	0.0130	322.215	– 4.0401
	Oil B			0.0164	304.639	
3	Oil A	1400	1.5	0.0106	375.917	– 3.8696
	Oil B			0.0131	355.413	
4	Oil A	1000	2.1	0.0169	268.512	2.0940
	Oil B			0.0150	253.866	
5	Oil A	1200	2.1	0.0119	322.215	– 3.6931
	Oil B			0.0146	304.639	
6	Oil A	1400	2.1	0.0124	375.917	1.8119
	Oil B			0.0112	355.413	
7	Oil A	1000	2.7	0.0139	268.512	1.2687
	Oil B			0.0130	253.866	
8	Oil A	1200	2.7	0.0111	322.215	– 1.6670
	Oil B			0.0122	304.639	
9	Oil A	1400	2.7	0.0097	375.917	0.9677
	Oil B			0.0092	355.413	
					Average	– 0.91

$$\alpha = \frac{\ln\left(\frac{C_{ma}}{C_{mb}}\right)^{\alpha}}{\ln\left(\frac{Re_a}{Re_b}\right)}. \tag{24}$$

With Equation (24) the average value of α for a range of revolutions 1000 – 1400 rpm is – 0.91. The calculation for α is given in Table 4.

Froude number (*Fr*). The Froude number is the ratio of the inertia force to the gravitational force.

The test results for the worm gear rotates at a different speed with the same immersion depth and the same lubricant was used to calculate the exponent. All other test conditions remained the same except the worm speed. Thus, the exponent can be derived from the two different speed test results at the same geometry and direction of rotation and expressed in the term of the dimensionless churning torques. Here speed is more dominating than other parameters.

$$\left(\frac{C_{ma}}{C_{mb}}\right) = \left(\frac{Fr_a}{Fr_b}\right)^{\beta}, \tag{25}$$

Table 5. Calculation of β at lubricant temperature 40 °C

No.	Type of oil	Speed, rpm	*V*, l	C_m	*Fr*	β
1	Oil A	1000	1.5	0.0164	22.3572	– 0.605
		1200		0.0132	32.1944	
2	Oil A	1000	1.5	0.0164	22.3572	– 0.640
		1400		0.0107	43.8202	
3	Oil A	1200	1.5	0.0132	32.1944	– 0.681
		1400		0.0107	43.8202	
4	Oil A	1000	2.1	0.0169	22.3572	– 0.976
		1200		0.0119	32.1944	
5	Oil A	1000	2.1	0.0169	22.3572	– 0.459
		1400		0.0124	43.8202	
6	Oil A	1200	2.1	0.0119	32.1944	0.151
		1400		0.0124	43.8202	
7	Oil A	1000	2.7	0.0169	22.3572	– 0.976
		1200		0.0119	32.1944	
8	Oil A	1000	2.7	0.0169	22.3572	– 0.459
		1400		0.0124	43.8202	
9	Oil A	1200	2.7	0.0119	32.1944	0.151
		1400		0.0124	43.8202	
10	Oil C	1000	1.5	0.0029	22.3572	0.055
		1200		0.0029	32.1944	
11	Oil C	1000	1.5	0.0029	22.3572	0.092
		1400		0.0031	43.8202	
12	Oil C	1200	1.5	0.0029	32.1944	0.136
		1400		0.0031	43.8202	
13	Oil C	1000	2.1	0.0028	22.3572	– 0.457
		1200		0.0024	32.1944	
14	Oil C	1000	2.1	0.0028	22.3572	– 0.168
		1400		0.0025	43.8202	
15	Oil C	1200	2.1	0.0024	32.1944	0.173
		1400		0.0025	43.8202	
16	Oil C	1000	2.7	0.0029	22.3572	– 0.280
		1200		0.0026	32.1944	
17	Oil C	1000	2.7	0.0029	22.3572	– 0.301
		1400		0.0023	43.8202	
18	Oil C	1200	2.7	0.0026	32.1944	– 0.326
		1400		0.0023	43.8202	
19	Oil B	1000	1.5	0.0172	22.3572	– 0.142
		1200		0.0163	32.1944	
20	Oil B	1000	1.5	0.0172	22.3572	– 0.403
		1400		0.0131	43.8202	
21	Oil B	1200	1.5	0.0163	32.1944	– 0.712
		1400		0.0131	43.8202	
22	Oil B	1000	2.1	0.0150	22.3572	– 0.085
		1200		0.0146	32.1944	
23	Oil B	1000	2.1	0.0150	22.3572	– 0.436
		1400		0.0112	43.8202	
24	Oil B	1200	2.1	0.0146	32.1944	– 0.850
		1400		0.0112	43.8202	
25	Oil B	1000	2.7	0.0130	22.3572	– 0.171
		1200		0.0122	32.1944	
26	Oil B	1000	2.7	0.0130	22.3572	– 0.508
		1400		0.0092	43.8202	
27	Oil B	1200	2.7	0.0122	32.1944	– 0.907
		1400		0.0092	43.8202	
					Average	– 0.36

with the index "a" for lower speed and index "b" for higher speed, all other variables concerning gear pair, lubricant type, lubricant volume and lubricant temperature did not change. With Equation (25) the average value of β for a range of revolutions 1000 – 1400 rpm is – 0.36. The calculation for β is given in Table 5.

The ratio of immersion depth to centre distance (*h*/*X*). The test results for the worm gear submerged at the different static heads with rotating speed and the same lubricant were used to calculate the exponent. The centre distance is constant (75 mm) for both gear pairs.

$$\left(\frac{C_{ma}}{C_{mb}}\right)=\left(\frac{\frac{h_a}{X}}{\frac{h_b}{X}}\right)^{\gamma}, \tag{26}$$

with the index "a" for the lower static head and index "b" for the higher static head, all other variables concerning gear pair, lubricant type, worm speed and lubricant temperature did not change. With Equation (26) the average value of γ for the range of speed 1000 – 1400 rpm and the defined range of static head is – 0.13. The calculation for γ is given in Table 6.

The ratio of lubricant volume to centre distance (V/X^3). The test results for the submerged worm gear at different lubricant volumes with the same rotating speed and same lubricant were used to calculate the exponent. The centre distance is constant (75 mm) for both gear pairs.

Table 6. Calculation of γ at lubricant temperature 40 °C

No.	Type of oil	Speed, rpm	*h*, mm	C_m	*h*/*X*	γ
1	Oil A	1000	40	0.0164	0.5333	0.042
			80	0.0169	1.0667	
2	Oil A	1200	40	0.0132	0.5333	– 0.153
			80	0.0119	1.0667	
3	Oil A	1400	40	0.0107	0.5333	0.218
			80	0.0124	1.0667	
4	Oil A	1000	40	0.0164	0.5333	– 0.136
			135	0.0139	1.8000	
5	Oil A	1200	40	0.0132	0.5333	– 0.141
			135	0.0111	1.8000	
6	Oil A	1400	40	0.0107	0.5333	– 0.077
			135	0.0097	1.8000	
7	Oil A	1000	80	0.0169	1.0667	– 0.372
			135	0.0139	1.8000	
8	Oil A	1200	80	0.0119	1.0667	– 0.126
			135	0.0111	1.8000	
9	Oil A	1400	80	0.0124	1.0667	– 0.467
			135	0.0097	1.8000	
					Average	– 0.13

Table 7. Calculation of δ at lubricant temperature 40 °C

No.	Type of oil	Speed, rpm	*V*, m^3	C_m	V/X^3	δ
1	Oil A	1000	0.0015	0.01644	3.5556	0.0866
			0.0021	0.01692	4.9778	
2	Oil A	1200	0.0015	0.01318	3.5556	−0.3151
			0.0021	0.01186	4.9778	
3	Oil A	1400	0.0015	0.01068	3.5556	0.4482
			0.0021	0.01242	4.9778	
4	Oil A	1000	0.0015	0.01644	3.5556	−0.2820
			0.0027	0.01393	6.4000	
5	Oil A	1200	0.0015	0.01318	3.5556	−0.2926
			0.0027	0.01110	6.4000	
6	Oil A	1400	0.0015	0.01068	3.5556	−0.1595
			0.0027	0.00973	6.4000	
7	Oil A	1000	0.0021	0.01692	4.9778	−0.7755
			0.0027	0.01393	6.4000	
8	Oil A	1200	0.0021	0.01186	4.9778	−0.2626
			0.0027	0.01110	6.4000	
9	Oil A	1400	0.0021	0.01242	4.9778	−0.9732
			0.0027	0.00973	6.4000	
					Average	– 0.28

$$\left(\frac{C_{ma}}{C_{mb}}\right)=\left(\frac{\frac{V_a}{X^3}}{\frac{V_b}{X^3}}\right)^{\delta}, \tag{27}$$

with the index “a” for lower volume and index “b” for higher volume, all other variables concerning gear pair, lubricant type, worm speed and lubricant temperature did not change. With Equation (27) the average value of δ for the range of speed 1000 – 1400 rpm and the defined range of static head is – 0.28. The calculation for δ is given in Table 7.

Reduction ratio (Ø). The test results for the different geometry of worm gear or different reduction ratios with the same rotating speed and lubricant properties.

$$\left(\frac{C_{ma}}{C_{mb}}\right)=\left(\frac{Ø_a}{Ø_b}\right)^{\varepsilon}, \tag{28}$$

with the index “a” for a higher reduction ratio and index “b” for a lower reduction ratio, all other variables concerning lubricant type, worm speed and lubricant temperature did not change. With Equation (28) the average value of ε for the range of speed 1000 – 1400 rpm and the defined range of static head is – 0.08. The calculation for ε is given in Table 8.

Table 8. Calculation of ε at lubricant volume 2.7 litre

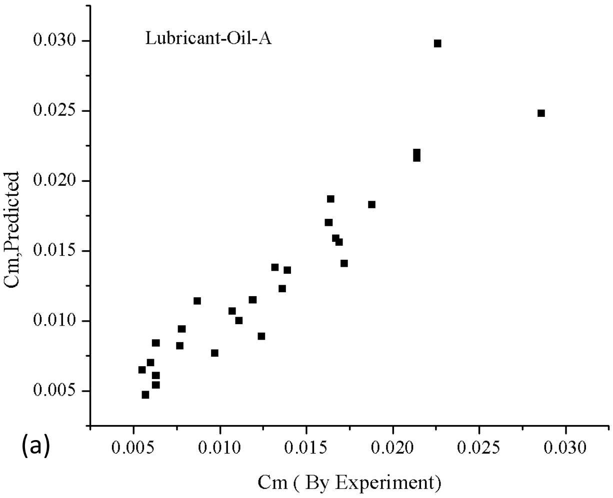
No.	Type of oil	Speed, rpm	Temperature, °C	C_m	Ø	ε
1	Oil A	1000	40	0.0139	30	– 0.0807
				0.0147	15	
2	Oil A	1200	40	0.0111	30	– 0.1244
				0.0121	15	
3	Oil A	1400	40	0.0097	30	– 0.1550
				0.0108	15	
4	Oil A	1000	50	0.0077	30	– 0.1426
				0.0085	15	
5	Oil A	1200	50	0.0063	30	– 0.0888
				0.0067	15	
6	Oil A	1400	50	0.0054	30	– 0.0780
				0.0057	15	
					Average	– 0.08

The following Equation (29) is developed by putting all the coefficients in Equation (22), which is known as an empirical equation of churning power loss for worm gear.

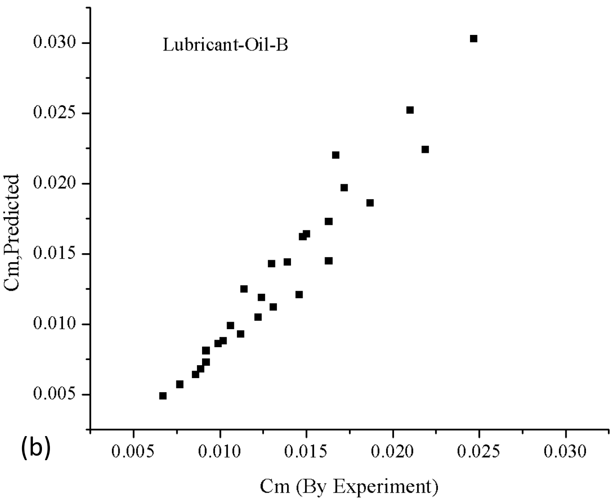
$$C_m = 17.08\left(\frac{h}{X}\right)^{-0.13}\left(\frac{V}{X^3}\right)^{-0.28} Re^{-0.91} Fr^{-0.38} \varnothing^{-0.08} \quad (29)$$

3. Results and discussion

Equation (29) is applied to all experiments of oil A and all experiments of oil B and compared with observed dimensionless torque Cm as shown in Figure 3.



(a)



(b)

Figure 3. Comparison between observed and predicted values of dimensionless torque C_m for: (a) oil A and (b) oil B

The values at the *x*-axis indicate the measured values of the experiment and those at the *y*-axis are the calculated values from the formula presented in this section. The scattered graph shows a good combination of the experiment values and predicted values. The variables of lubricant immersion depth, lubricant volume, centre distance and Reynolds and Froude number were considered and the value of the root mean square error (RMSE) was calculated for all the experiments. RMSE for oil A is 0.00213238 and RMSE for oil B is 0.002252324. The root mean square error is only 0.2 % for oil A and oil B which shows very good agreement for predicted and experimental values of dimensionless torque C_m. The RMSE shows a negligible discrepancy between the experiment and predicted values.

3.1 Confirmation test

Equation (29) is the result of the dimensional analysis of experimental values. This equation is derived by considering only oil A and oil B as a lubricant. To validate this developed model, oil C is introduced. This equation is also validated by designing a new test matrix as shown in Table 9.

Table 9. Test matrix to validate the empirical equation

Factor	Level
Worm speed	900, 1100 and 1300 rpm
Type of oil	Oil C
Oil volume	2.7 litre
Direction of rotation	Forward (clockwise of worm)

The speed of the gear and type of lubricant is revised. The developed mathematical model is applied to the values of the newly designed test matrix to obtain the predicted dimensionless churning torque (C_m). The results of the confirmation experiments with predicted results are shown in Figure 4 in the form of a scattered graph.

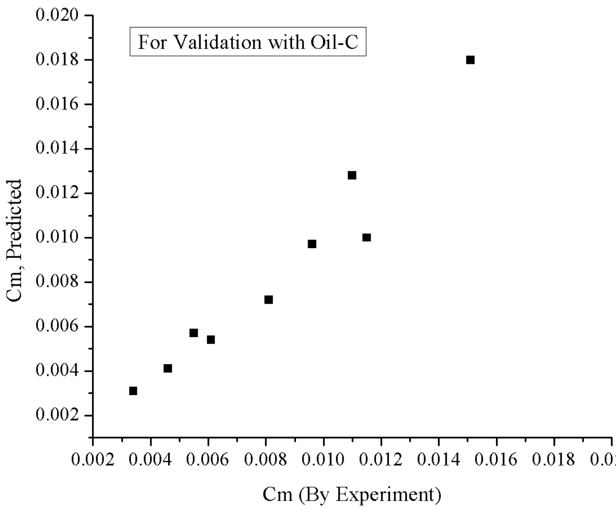


Figure 4. Comparison between observed and predicted values of dimensionless torque C_m for confirmation

A very good agreement between the predicted and observed dimensionless torque for the confirmation test is shown in Figure 4. The root mean square error for the confirmation test is 0.00131, as shown in Table 10. The RMSE is only 0.1 % showing good agreement between the measured and experiment values.

Table 10. The root mean square error calculation for the confirmation test

No.	C_m (by experiment)	C_m (predicted)	Residual (error)	Square of residual
1	0.0151	0.0180	– 0.0029	8.41E–06
2	0.0110	0.0128	– 0.0018	3.24E–06
3	0.0096	0.0097	– 0.0001	1E–08
4	0.0115	0.0100	0.0015	2.25E–06
5	0.0081	0.0072	0.0009	8.1E–07
6	0.0061	0.0054	0.0007	4.9E–07
7	0.0055	0.0057	– 0.0002	4E–08
8	0.0046	0.0041	0.0005	2.5E–07
9	0.0034	0.0031	0.0003	9E–08
			Average RMSE	0.00131

4. Conclusion

A mathematical model was proposed in this study to measure the dimensionless churning torque. This was obtained through a dimensional analysis based on the experiment result. Specific characteristics of the proposed model are given below.

- It counts the churning power loss of the worm shaft and worm gear in the mesh position only.
- This model is valid for both orientations of the worm shaft and worm gear (worm shaft at the top and worm shaft at the bottom).
- This model is valid for both directions (clockwise and anticlockwise).
- This model is proposed for any type of liquid lubricant (mineral and synthetic).

It has been validated by many experiments from a specific test rig. The validation shows the average error between the experiment and prediction is only 0.0001. The root mean square error is only 0.1 % showing good agreement between the predicted and experimental values.

The limitations of the model are given below.

- The model is valid for splash lubrication only; jet lubrication is not considered.
- The model developed in this research work is valid for worm gearbox.
- Pocketing loss and windage loss are not considered in this model.

Future scope: The mathematical model can be fabricated by considering jet lubrication. The same mathematical model can be formulated for pocketing loss and windage loss.

Nomenclature

b	gear face width, m
C_{ch}	drag (churning) torque, Nm
C_m	dimensionless drag torque
F_d	drag force, N
Fr	Froude number
g	gravitational acceleration, m/s^2
h	immersion depth, m
P_{bc}	churning power loss of bearings, W
P_{bf}	friction power loss of bearings, W
P_s	seals power loss, W
P_{wc}	churning power loss of worm gear, W
P_{wf}	friction power loss of worm gear, W
R	radius of worm shaft, m
r	gear pitch radius, m
Re	Reynolds number
R_p	pitch radius of gear, m
S_m	immersed surface area, m^2
u	speed ratio
V	oil volume, m^3
v	velocity of the rotating object, m/s
V_o	submerged volume, m^3
X	centre distance, m
$Ø$	reduction ratio
α	coefficient of Reynolds number
β	coefficient of Froude number
γ	coefficient of the ratio of immersion depth to the pitch circle radius
ΔP	additional power loss, W
δ	coefficient of the ratio of total volume to the cube of the pitch circle radius
ε	coefficient of reduction ratio
μ	oil dynamic viscosity, Pas
ρ	oil density, kg/m^3
ψ	dimensionless constant
ω	angular velocity, s^{-1}

References

[1] H.G. Chothani, K.D. Maniya, Experimental investigation of churning power loss of single start worm gear drive through optimization technique, Materials Today: Proceedings, Vol. 28, No. 4, 2020, pp. 2031-2038, DOI: 10.1016/j.matpr.2019.12.365

[2] S. Seetharaman, A. Kahraman, M.D. Moorhead, T.T. Petry-Johnson, Oil churning power losses of a gear pair: Experiments and model validation,

Journal of Tribology, Vol. 131, No. 2, 2009, Paper 022202, DOI: 10.1115/1.3085942

[3] A.S. Kolekar, A.V. Olver, A.E. Sworski, F.E. Lockwood, Windage and churning effects in dipped lubrication, Journal of Tribology, Vol. 136, No. 2, 2014, Paper 021801, DOI: 10.1115/1.4025992

[4] X. Hu, Y. Jiang, C. Luo, L. Feng, Y. Dai, Churning power losses of a gearbox with spiral bevel geared transmission, Tribology International, Vol. 129, 2019, pp. 398-406, DOI: 10.1016/j.triboint.2018.08.041

[5] R.W. Mann, C.H. Marston, Friction drag on bladed disks in housings as a function of Reynolds number, axial and radial clearance, and blade aspect ratio and solidity, Journal of Basic Engineering, Vol. 83, No. 4, 1961, pp. 719-723, DOI: 10.1115/1.3662307

[6] J.W. Daily, R.E. Nece, Chamber dimension effects on induced flow and frictional resistance of enclosed rotating disks, Journal of Basic Engineering, Vol. 82, No. 1, 1960, pp. 217-230, DOI: 10.1115/1.3662532

[7] A.S. Terekhov, Hydraulic losses in gearboxes with oil immersion, Russian Engineering Research, Vol. 55, No. 5, 1975, pp. 7-11.

[8] R.J. Boness, Churning losses of discs and gears running partially submerged in oil, Proceedings of the 1989 International Power Transmission and Gearing Conference, Vol. 1, 25-28.04.1989, Chicago, USA, pp. 355-359.

[9] P. Luke, A.V. Olver, A study of churning losses in dip-lubricated spur gears, Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, Vol. 213, No. 5, 1999, pp. 337-346, DOI: 10.1243/0954410991533061

[10] C. Changenet, P. Velex, A model for the prediction of churning losses in geared transmissions – Preliminary results, Journal of Mechanical Design, Vol. 129, No. 1, 2007, pp. 128-133, DOI: 10.1115/1.2403727

[11] S. Seetharaman, A. Kahraman, Load-independent spin power losses of a spur gear pair: Model formulation, Journal of Tribology, Vol. 131, No. 2, 2009, Paper 022201, DOI: 10.1115/1.3085943

[12] S. Laruelle, C. Fossier, C. Changenet, F. Ville, S. Koechlin, Experimental investigations and analysis on churning losses of splash lubricated spiral bevel gears, Mechanics & Industry, Vol. 18, No. 4, 2017, Paper 412, DOI: 10.1051/meca/2017007

[13] Q. Peng, L. Gui, Z. Fan, Numerical and experimental investigation of splashing oil flow in a hypoid gearbox, Engineering Applications of Computational Fluid Mechanics, Vol. 12, No. 1, 2018, pp. 324-333, DOI: 10.1080/19942060.2018.1432506

[14] Y. Ariura, T. Ueno, T. Sunaga, S. Sunamoto, The lubricant churning loss in spur gear systems, Bulletin of JSME, Vol. 16, No. 95, 1973, pp. 881-892, DOI: 10.1299/jsme1958.16.881

[15] J. Polly, D. Talbot, A. Kahraman, A. Singh, H. Xu, An experimental investigation of churning power losses of a gearbox, Journal of Tribology, Vol. 140, No. 3, 2018, Paper 031102, DOI: 10.1115/1.4038412

[16] P.M.T. Marques, C.M.C.G. Fernandes, R.C. Martins, J.H.O. Seabra, Power losses at low speed in a gearbox lubricated with wind turbine gear oils with special focus on churning losses, Tribology International, Vol. 62, 2013, pp. 186-197, DOI: 10.1016/j.triboint.2013.02.026

[17] G. Leprince, C. Changenet, F. Ville, P. Velex, Investigations on oil flow rates projected on the casing walls by splashed lubricated gears, Advances in Tribology, Vol. 2012, 2012, Paper 365414, DOI: 10.1155/2012/365414

[18] C.M.C.G. Fernandes, P.M.T. Marques, R.C. Martins, J.H.O. Seabra, Gearbox power loss. Part III: Application to a parallel axis and a planetary gearbox, Tribology International, Vol. 88, 2015, pp. 317-326, DOI: 10.1016/j.triboint.2015.03.029

[19] S.I. Jeon, Improving Efficiency in Drive Lines: An Experimental Study on Churning Losses in Hypoid Axle, PhD thesis, Department of Mechanical Engineering, Imperial College of Science, Technology and Medicine London, London, 2010, DOI: 10.25560/5861