Tribology and Materials
Vol. 1, No. 4, 2022, pp. 145-149

ISSN 2812-9717
https://doi.org/10.46793/tribomat.2023.018

Numerical simulation of two-dimensional crack propagation using stretching finite element method by Abaqus

Mohammed BENTAHAR [1,*], **Habib BENZAAMA** [2]

[1] Faculty of Technology, University of Saida, Saida, Algeria
[2] National Polytechnic School of Oran, Oran, Algeria
*Corresponding author: bentahae@yahoo.fr

Keywords

2D crack propagation
FEM
SFEM
XFEM
Abaqus
SIF

History

Received: 14-12-2022
Revised: 29-12-2022
Accepted: 30-12-2022

Abstract

Fatigue is a phenomenon that appears in items subjected to cyclic loads. Thus, rupture and partial oxidation are initiated from the beginning of the process. For this, the study of such a problem of material in fracture mechanics is based on the numerical analysis of the characteristics of a crack. In this article, we propose a modelling of crack propagation by a new method of stretching the mesh by 2D finite element in mixed mode, based on the creation of the elements through a program in the computer language Fortran. The parametric mesh with 4 nodes (CPE4) was created to make the simulation and to characterise the stress intensity factors, by the Abaqus computer code. For this, the validation of stretching finite element method (SFEM) results is done by other methods: extended finite element method (XFEM) and analytical method to simulate the crack propagation. The stress intensity factor (SIF) is an essential parameter of this study. Two possibilities for determining the SIF have been retained: one by the numerical method of our choice and the other by the analytical method. Parameters to characterise the stress state at the crack front K_{I} and K_{II} were evaluated in two stages one by the crack length and the other by the *a*/*c* ratio.

1. Introduction

Fracture mechanics is a domain very broad, very big and very complex, particularly in the problems of crack propagation. The last is a complex numerical problem since it requires the following of the geometry of the crack during the time. It rests on the principles of fracture mechanics, specifically by calculating the SIF in various modes. Many analytical formulas exist to determine the SIF [1]. However, these expressions are often developed for simple cases of geometry and solicitations, Tada et al. [2]. As soon as the geometry or the solicitation becomes more complex, it becomes necessary to use the principle of functions of weight superposition.

Several authors have developed numerical methods for modelling the propagation of cracks; one can also cite the work of Chan et al. [3]. Linear fracture mechanics remains today the most used in practice. Among always the most recent references to the finite element method (FEM), Mueller and Maugin [4] studied the case of crack propagation in 2D. The representation of the singular field around the crack tip can be improved by the use of the so-called singular elements of Barsoum [5]. Various numerical methods from continuum mechanics were adapted to study the behaviour of materials up to rupture, for example, the finite element method (FEM), the extended finite element method (XFEM), etc.

At first, we are interested in problems of the fracture mechanics treated by the method of classical finite elements, then by XFEM introduced in Belytschko and Black [6] and Moës et al. [7]. The study of cracking by FEM began in the sixties. On the other hand, the advantage of these enriched finite elements is that the SIF can be calculated directly from the results. Another method based on the classic FEM is called the extended finite element method (XFEM) or the generalized finite element method (GFEM). This method is an extension of the FEM and has been applied to the problems of fracture mechanics since 1999. Crack propagation in this method is simulated, replacing the new elements crossed by a crack with a special element presented in the 2D method, Moës et al. [7]. The latter is based on the concept of the partition of unity method (PUM) proposed by Babuška and Melenk [8].

In recent years several works have been published. Meor Ahmad et al. [9] studied a numerical strategy for the predictive modelling of creep in tensile tests of a rectangular plate containing a crack using the XFEM. Bentahar et al. [10] used the 2D XFEM first mode I, to model crack propagation and energy evaluation at the crack tip. On the other hand, this method has been used by Chen et al. [11] for numerical prediction and to accurately simulate fatigue life and cyclic overload effect of crack propagation by the Abaqus software of Python code. Regarding crack growth, Lee et al. [12] developed a new advanced method (AI-FEM) by the simulation program, to calculate the exact stress intensity factor regarding arbitrary structures. Malekan et al. [13] presented a new freely distributed 2D plug-in to simulate fatigue crack growth by FE Abaqus code. On the other hand, Alshoaibi and Fageehi [14] have numerically simulated by the Ansys calculation code for the calculation of the stress intensity factor of a material with linear elastic properties, and the predictive pavement cracking. Alkaissi [15] used the Abaqus software FEM to analyse crack propagation. In our work, we propose a new method called finite elements classic SFEM (stretching finite element method) based on the creation of 2D elements and calculated in the Abaqus code.

2. Fracture mechanics

2.1 Crack propagation

The law of 2D crack propagation was proposed by Paris and Erdogan [16]. Equation (1) explains the relationship between the variation of SIF and the propagation speed.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \mathrm{C}(\Delta K)^{\mathrm{m}}, \tag{1}$$

where a is the crack length, N is the number of loading cycles, C and m are the material parameters and ΔK is the variation of the stress intensity factor.

2.2 Two-dimensional criterion by Richard

This criterion was empirically developed by Richard [17,18]. The comparative stress intensity factor K_{V} is defined by:

$$K_{\mathrm{V}} = \frac{K_{\mathrm{I}}}{2} + \frac{1}{2}\sqrt{K_{\mathrm{I}}^2 + 5.366\,K_{\mathrm{II}}^2} = K_{\mathrm{Ic}}. \tag{2}$$

Under monotonic uniaxial loading, crack growth occurs when the stress intensity factor K_{I} near the crack tip reaches its critical value K_{Ic}. Then, the following criterion can be written:

$$K_{\mathrm{I}} = K_{\mathrm{Ic}}. \tag{3}$$

Factor K_{V} depends on the stress intensity factors K_{I} and K_{II}. Note that unstable crack growth occurs if K_{V} exceeds fracture toughness K_{Ic}. This criterion has an excellent approximation to the yield stress surface of the maximum tangential stress criterion of Erdogan and Sih [19]. The twist angle of the crack θ can be determined by:

$$\theta = \mp\left[140^{\circ}\frac{|K_{\mathrm{II}}|}{|K_{\mathrm{I}}|+|K_{\mathrm{II}}|} - 70^{\circ}\left(\frac{|K_{\mathrm{II}}|}{|K_{\mathrm{I}}|+|K_{\mathrm{II}}|}\right)^2\right]. \tag{4}$$

First of all, the stress intensity factor $K_{\mathrm{II}} > 0$, the angle $\theta < 0$ and $K_{\mathrm{I}} > 0$. There are other criteria, which are based on the rate of energy restitution like the Nuismer criterion [20] or Amestoy criterion [21].

2.3 Different cracking modes

The presence of defects may be in the form of internal cracks or surface cracks. Fracture mechanics analysis correlates parameters from loading, geometry, and material. Then, it predicts the conditions under which a crack can propagate and possibly lead to the complete failure of the structure.

In the continuous medium, cracking is an irreversible separation phenomenon. Three main modes of cracking can be distinguished (Fig. 1).

- aperture mode (mode I),
- shear mode (mode II) and
- anti-plane shear mode (mode III).

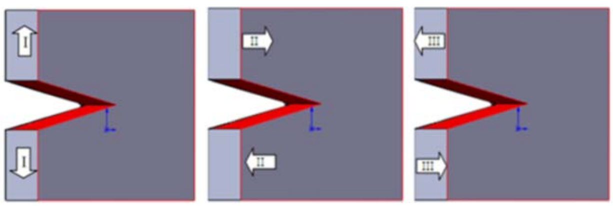

Figure 1. Illustration of fracture cracking modes [22]

2.4 Illustration of the stress distribution at the crack tip

Figure 2 illustrates the stress field near the point of a crack with the polar coordinates (r, θ) in the vicinity of the crack tip. The general equation of the stress field in 2D near the crack tip, defined by the K stress intensity factor is given by Tada et al. [2]:

$$\sigma_{i,j}^{\mathrm{I,II}}(r,\theta)=\frac{K_{\mathrm{I,II}}}{\sqrt{2\pi r}}f_{ij}(\theta), \tag{5}$$

where $K_{\mathrm{I,\,II}}$ is SIF in modes I and II and $\sigma_{i,j}^{\mathrm{I,II}}$ is the stress field associated with mode I.

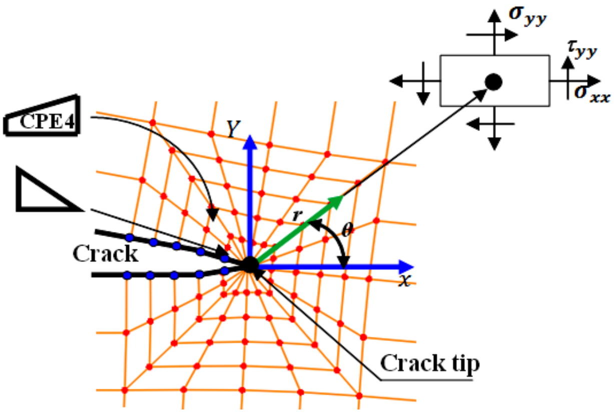

Figure 2. Stress field in the crack front vicinity in our model [23]

Factor K_{I} is the one that contributes the most to the propagation of fatigue cracks; these tend to spread following the direction perpendicular to the maximum tangential stress in its end, Erdogan and Sih [19].

$$\sigma_{xx}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{yy}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy}=\frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}. \tag{6}$$

2.5 Analytical calculation of stress intensity factor

The analytical stress intensity factor K_{I} for this problem is given by Ewalds and Wanhill [1] as:

$$K_{\mathrm{I}}=\mathrm{Y}\sigma\sqrt{a\pi}, \tag{7}$$

where Y is the correction factor given by

$$\mathrm{Y}=1.12-0.23\left(\frac{a}{c}\right)+10.6\left(\frac{a}{c}\right)^2-$$
$$-21.7\left(\frac{a}{c}\right)^3+30.4\left(\frac{a}{c}\right)^4. \tag{8}$$

The non-trivial solution of θ is given by

$$K_{\mathrm{I}}\sin\theta+K_{\mathrm{II}}(3\cos\theta-1)=0. \tag{9}$$

3. Numerical model (mesh)

In the present question, we consider the plate to be rectangular, with dimensions (B × C); length and width, respectively. The mesh was generated by the Fortran program. The plate was of ASTM A36 steel with a modulus of elasticity E = 200 GPa and Poisson's ratio v = 0.26. The finite element Abaqus code was used to calculate the various SIF and crack propagation angles. A uniform tensile stress σ = 200 MPa was applied to both surfaces (Fig. 3).

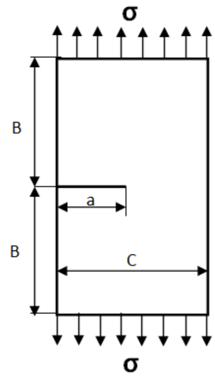

Figure 3. Specimen with crack

Figure 4 shows the two compared models. Figure 4a presents the model of the SFEM and Figure 4b illustrates the model of the XFEM, for the different crack propagations. Thus, this figure presents the crack path, by the angle of inclination (α) concerning the two methods.

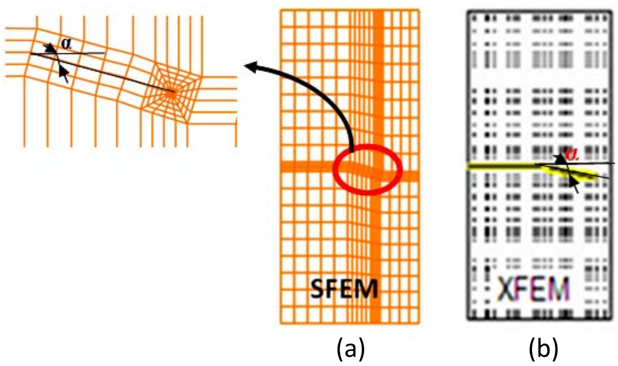

Figure 4. Comparison: (a) SFEM model for 5 crack propagations and (b) XFEM model

4. Results and discussion

Figure 5 explains the comparison of the stress intensity factors in mixed mode, of parametric mesh crack propagation between the three methods: XFEM, SFEM and the analytical method proposed by [1]. According to the ratio (a/c), the latter takes the values 0.5, 0.6, 0.7 and 0.8. It can be seen that the results obtained are very proportional and acceptable.

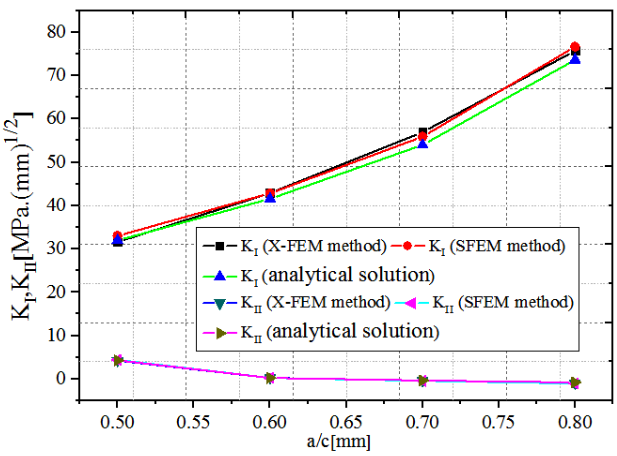

Figure 5. Comparison of SIF between three methods (SFEM, XFEM and the analytical solution) as a function of *a*/*c*

Figure 6 presents the relationship between the stress intensity factors and the crack length (*a*) throughout the length of the crack propagation of the parametric mesh. Thus, the variation between the different methods we studied in our work for them to have the proportionality of the results. K_I present a remarkable evolution between each crack propagation and each inclination angle α. On the other hand, we can see that K_I is always positive and K_{II} is negative.

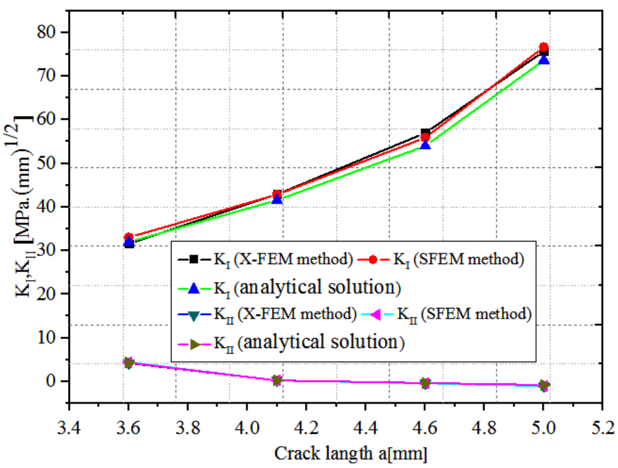

Figure 6. Comparison of SIF between three methods (SFEM, XFEM and the analytical solution) depending on the crack length (*a*)

Figure 7 illustrates the relationship between the angle of inclination α and the stress intensity factors in mixed mode, for XFEM and the analytical method proposed by [1]. In this comparison, it could be noticed that most of the α values are < 0. It is observed that the variation of stress intensity factors with the angle of inclination is very small, except when the angle α is between − 0.719° and − 0.7127°.

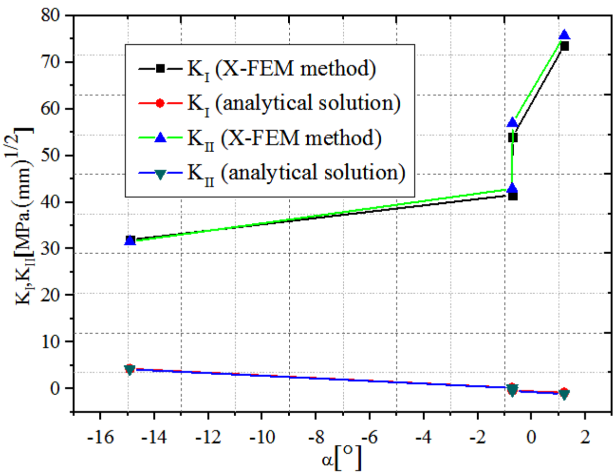

Figure 7. Evolution of K_I and K_{II} according to the inclination angle (α) for XFEM and the reference method proposed by [1]

5. Conclusion

The SFEM was used to model the problem of the initial crack of length a = 3.5 mm. On the other hand, the Abaqus software was used to determine the angle of inclination at each crack propagation, and in addition, the stress intensity factors under the mixed mode with the SFEM and XFEM, which are numerical methods. In order to obtain a better approximation of the field near the crack tip, special quarter-point finite elements are used.

Numerical calculations by the finite element method of stretching of the mesh (SFEM) show that this model can correctly describe the stress field and deformation near the crack tip. The results obtained between the three methods are very proportional. CPE4 elements were used to do the modelling by Abaqus.

The stress intensity factor (SIF) is an essential parameter of this study. Two possibilities for determining the SIF have been retained: one by the numerical method of our choice (SFEM) and the other by the analytical method.

References

[1] H.L. Ewalds, R.J.H. Wanhill, Fracture Mechanics, Edward Arnold, London, 1984.

[2] H. Tada, P.C. Paris, G.R. Irwin, The Stress Analysis of Cracks Handbook, ASME Press, New York, 2000, DOI: 10.1115/1.801535

[3] S.K. Chan, I.S. Tuba, W.K. Wilson, On the finite element method in linear fracture mechanics,

Engineering Fracture Mechanics, Vol. 2, No. 1, 1970, pp. 1-17, DOI: 10.1016/0013-7944(70)90026-3

[4] R. Mueller, G.A. Maugin, On material forces and finite element discretizations, Computational Mechanics, Vol. 29, No. 1, 2002, pp. 52-60, DOI: 10.1007/s00466-002-0322-2

[5] R.S. Barsoum, On the use of isoparametric finite elements in linear fracture mechanics, International Journal for Numerical Methods in Engineering, Vol. 10, No. 1, 1976, pp. 25-37, DOI: 10.1002/nme.1620100103

[6] T. Belytschko, T. Black, Elastic crack growth in finite elements with minimal remeshing, International Journal for Numerical Methods in Engineering, Vol. 45, No. 5, 1999, pp. 601-620, DOI: 10.1002/(SICI)1097-0207(19990620)45:5<601::AID-NME598>3.0.CO;2-S

[7] N. Moës, J. Dolbow, T. Belytschko, A finite element method for crack growth without remeshing, International Journal for Numerical Methods in Engineering, Vol. 46, No. 1, 1999, pp. 131-150, DOI: 10.1002/(SICI)1097-0207(19990910)46:1<131::AID-NME726>3.0.CO;2-J

[8] I. Babuška, J. M. Melenk, The partition of unity method, International Journal for Numerical Methods in Engineering, Vol. 40, No. 4, 1997, pp. 727-758, DOI: 10.1002/(SICI)1097-0207(19970228)40:4<727::AID-NME86>3.0.CO;2-N

[9] M.I. Meor Ahmad, M.A. Mohd Sabri, M.F. Mat Tahir, N.A. Abdullah, Predictive modelling of creep crack initiation and growth using extended finite element method (XFEM), Frattura ed Integrità Strutturale, Vol. 16, No. 61, 2022, pp. 119-129, DOI: 10.3221/IGF-ESIS.61.08

[10] M. Bentahar, H. Benzaama, N. Mahmoudi, Numerical modeling of the crack propagation for the evolution of the strain energy ALLSE by the X-FEM method, Revue des Matériaux et Energies Renouvelable, Vol. 5, No. 2, 2021, pp. 24-31.

[11] Z. Chen, H. Bao, Y. Dai, Y. Liu, Numerical prediction based on XFEM for mixed-mode crack growth path and fatigue life under cyclic overload, International Journal of Fatigue, Vol. 162, 2022, Paper 106943, DOI: 10.1016/j.ijfatigue.2022.106943

[12] G.-B. Lee, S.-H. Park, Y.-Y. Jang, N.-S. Huh, S.-H. Park, N.-H. Park, J. Park, Development of automatic crack growth simulation program based on finite element analysis, Applied Sciences, Vol. 12, No. 6, 2022, Paper 3075, DOI: 10.3390/app12063075

[13] M. Malekan, A. Khosravi, L. St-Pierre, An Abaqus plug-in to simulate fatigue crack growth, Engineering with Computers, Vol. 38, No. 4, 2022, pp. 2991-3005, DOI: 10.1007/s00366-021-01321-x

[14] A.M. Alshoaibi, Y.A. Fageehi, Finite element simulation of a crack growth in the presence of a hole in the vicinity of the crack trajectory, Materials, Vol. 15, No. 1, 2022, Paper 363, DOI: 10.3390/ma15010363

[15] Z.A. Alkaissi, Modelling of crack propagation in flexible pavement using X-FEM method, IOP Conference Series: Earth and Environmental Science, Vol. 961, 2022, Paper 012014, DOI: 10.1088/1755-1315/961/1/012014

[16] P. Paris, F. Erdogan, A critical analysis of crack propagation laws, Journal of Basic Engineering, Vol. 85, No. 4, 1963, pp. 528-533, DOI: 10.1115/1.3656900

[17] H.A. Richard, Bruchvorhersagen bei überlagerter Normal- und Schubbeanspruchung von Rissen [Fracture Predictions With Superimposed Normal and Shear Stresses of Cracks], VDI-Verlag, Düsseldorf, 1985 [in German].

[18] H.A. Richard, Safety estimation for construction units with cracks under complex loading, in H.P. Rossmanith (Ed.), Structural Failure, Product Liability and Technical Insurance, Inderscience Enterprises, Geneva, 1987, pp. 423-437.

[19] F. Erdogan, G.C. Sih, Closure to "Discussion of ''On the crack extension in plates under plane loading and transverse shear'", Journal of Basic Engineering, Vol. 85, No. 4, 1963, p. 527, DOI: 10.1115/1.3656899

[20] R.J. Nuismer, An energy release rate criterion for mixed mode fracture, International Journal of Fracture, Vol. 11, No. 2, 1975, pp. 245-250, DOI: 10.1007/BF00038891

[21] M. Amestoy, H.D. Bui, K. Dangvan, Analytic asymptotic solution of the kinked crack problem, in D. Francois (Ed.), Advances in Fracture Research, Pergamon Press, Oxford, 1980, pp. 107-113.

[22] M. Bentahar, Etude et Modélisation de Propagation de Fissure en Eléments Finis en Deux Dimensions [Study and Modelling of Crack Propagation in Finite Elements in Two Dimensions], PhD thesis, National Polytechnic School of Oran, Oran, 2017 [in French].

[23] M. Bentahar, H. Benzaama, M. Bentoumi, M. Mokhtari, A new automated stretching finite element method for 2D crack propagation, Journal of Theoretical and Applied Mechanics, Vol. 55, No. 3, 2017, pp. 869-881, DOI: 10.15632/jtam-pl.55.3.869