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Comparison of the theoretical and experimental coefficient of friction for the brake disc-brake pad system

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Abstract

Contact between the automotive brake pad and the disc is mathematically modelled to estimate the coefficient of friction (COF). The mathematical model is proposed for the prognosis of the COF of brake pad material, by considering the contact mechanics between the interfacing surface and their material properties. The Greenwood-Williamson contact model is applied for rough contact surfaces for the estimation of the real contact radius. A MATLAB program has been formulated for generating the surface of brake pad material by considering its material properties which aid in the analytical evaluation of the COF. The proposed model is further validated with experimentation on pin-on-disc apparatus, as it is considered suitable for friction pad product testing according to previous research. The 25 pins were fabricated as per the ASTM G99 test for testing under varying loads and speeds. The obtained results showed that the range of COF has been between 0.2 and 0.4. The investigation presents an analytical approach for estimating COF and contact radius for brake disc and brake pad, which can be used to design an efficient automotive brake disc-brake pad system under the given load and rotational speed. The artificial neural network (ANN) is modelled for predicting the values of the COF for brake disc-brake pad systems, which can be further used for determining the tribological properties of new friction materials and their compatibility for efficient brake systems.

1. Introduction

Automotive brake disc-brake pad systems must have a stable coefficient of friction during service life, good wear resistance, low thermal expansion properties and high thermal conductivity. Brake pads which are also known as friction pads transfer kinetic energy into heat during the application of a brake to slow down or stop a moving vehicle. The counterparts, i.e. brake disc realises the necessary brake moment or transmission of torsion [1-3]. The oldest and only one suitable to slow down or stop an automobile, within set deceleration and braking distance, is the mechanical braking system which is based on tribological principles. The automotive industry has great challenges regarding various vehicle components to increase performance as well as cost reduction in the area of the different manufacturing processes and safety. Brake system is one of the vital parts in the automobile from the safety and performance aspect. The brake system consists of metallic brake disc and brake pads in order to maintain a steady friction coefficient. The coefficient of friction (COF) of a material is dependent upon the counterface material, surface preparation and operating conditions.

To investigate the tribological behaviour of materials in contact with sliding motion mostly pin-on-disc apparatus is used. Also, this approach is particularly suited to study the existing relationships between wear and friction mechanisms and parameters like rotational speed, contact pressure and environmental conditions [1,2]. Studies of investigations on materials for brake systems, particularly for road vehicles [3-7] and trains [8-9], report the pin-on-disc results. The pin-on-disc is ideal for product certification of automotive friction pads and mainly to obtain design-oriented information. Even so, plain pin-on-disc testing is very useful to obtain focused information on the friction and wear mechanisms and their role in the tribological behaviour of real systems [6,7,10-13]. Additionally, considering the complexity of the formulation of friction materials for brake pads, it is paramount to have a reliable selection tool for the development of novel compositions [14,15].

Sliding friction between two dry contacting surfaces is often known as "Coulomb friction" after Charles Coulomb (1736-1806) [16]. Still, despite its everyday nature, the friction forces involved can usually only be estimated from previous experience and experimental evidence. Transfer films are coats on the surfaces in sliding contact in friction brakes that occurs as a result of the complex interaction between the pad and disc materials under high temperatures and pressures [17]. Model systems have been studied in which the materials and surfaces were scientifically controlled in the expectation that once the friction of such systems was understood, more and more complicated systems could be examined [16]. Thus, by using the theories for contact mechanics which considers the actual scenario of two contacting surfaces, an analytical model for the estimation of COF at the brake disc-brake pad interface is developed.

The early investigation in the field of contact mechanics was made by Greenwood and Williamson (GW). They discovered that many important properties of the contact are almost independent of the local asperity behaviour if the asperity height distribution in Gaussian. One surface is considered to have the combined roughness of the two original surfaces and the other is considered to be smooth. In the GW framework [18], the contact between two rough surfaces is modelled as a contact problem between a rigid flat surface and an elastic solid decorated with a randomly rough surface, which is further assumed to be an ensemble of non-interacting spherical asperities of the same curvature radius R.

It is worth noting that the idea underlying the GW's approach has also been generalised to establish models of rough surface with multiple representative shapes and curvature radii [19].

To conclude, the GW model is a basic model to calculate pressure between any two flat surfaces with consideration of their physical material properties. The proposed mathematical model is an approach to the application of the GW model on the brake disc-brake pad surfaces to calculate the interface pressure accurately.

According to friction theories, the friction coefficient between two surfaces is an intrinsic constant of the material, but also this is only for a particular speed line, contact pressure, lubrication state, etc. [6-8]. The analysis of the tribological behaviour of coupling materials (composite/iron cast materials) follows the evaluation of friction coefficient, depending on different loading parameters such as pressure and sliding speed. The present work proposes a theoretical model for the investigation of the friction coefficient by considering the material properties of the brake pad and disc. A MATLAB program has been developed to generate a rough surface and compute the height of asperities of the brake pad surface during contact interface, which assists in estimating the real radius of contact and further computation of COF. Experimentation on pin-on-disc was conducted and obtained data were compared with the analytically calculated results for validation of the proposed theoretical model. An artificial neural network (ANN) model based on the proposed mathematical model is structured and trained by experimentation on pin-on-disc for predicting tribological factors such as COF.

2. Analytical investigation of COF for disc brake

The methodology for the estimation of friction coefficient (μ) considering interface pressure on asperities of brake friction material at the contact interface of brake disc and pad is presented by Kshirsagar and Khairnar [20]. The pressure P_a on the asperities is given by considering contact radius (a) and load (W) with the total number of asperities as z on the brake pad surface.

$$P_a = \sum_{i=1}^{z} \frac{3W_i}{2\pi a_i^2}. \tag{1}$$

For determining the contact radius (a) at the brake disc-brake pad interface, contact mechanics theory is applied, in the form of the Greenwood-Williamson.

$$a = 2\sigma^2 \ln(Ra / 2\pi\sigma^4) + l, \tag{2}$$

where, a is the contact radius which varies according to the length of the surface in contact (l), roughness (Ra) and surface density (σ). Thus, by determining the contact radius real area of contact can be determined. To simplify the determination of contact radius a MATLAB program was used for generating a random Gaussian surface with given surface properties. The contact radius of the asperities was calculated by applying the fast Fourier transform on the obtained image in MATLAB, and the radius of asperities is obtained. This radius of asperities is further used for calculating the COF.

3. Experimental investigation of COF for disc-pad brake system

For the present experimentation, a pin-on-disc machine by Ducom Instruments is used as shown in Figure 1. A pin-on-disc tribometer consists of a stationary pin under an applied load in contact with a rotating disc. The coefficient of friction (COF) is determined by the ratio of the frictional force to the loading force on the pin. The machine is attached to a data acquisition system and Winducom 2010 software gives result values and graphs.

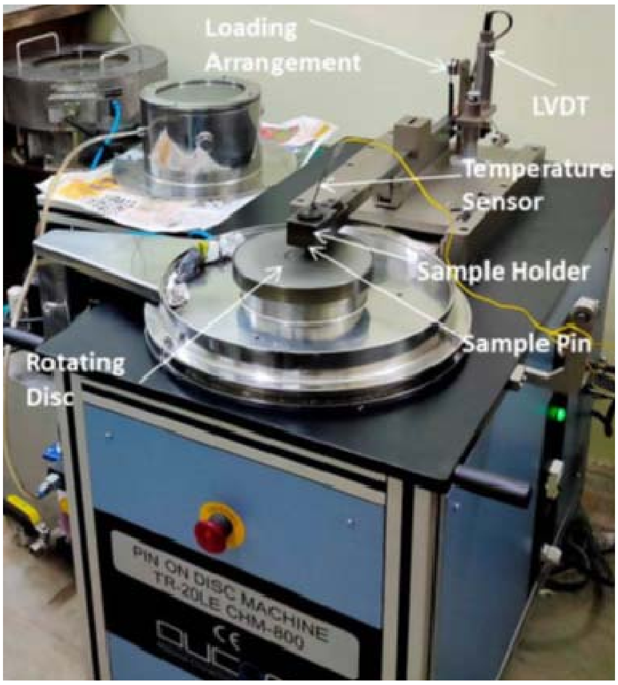

Figure 1. Pin-on-disc apparatus

The track diameter is set which means that the pin will be in contact at that circumference of the disc. The contact between the pin and disc for the experimentation is the area of contact. Using different loads, speeds and time, results for the COF are obtained. All the readings procured are shown on the Winducom 2010 software from which we obtain the values of friction force, COF, sliding speed and sliding distance. The graphs are obtained, which help to study the characteristics of the parameters obtained during the test. Following Table 1 represents the test parameters used in the experimentation.

Table 1. Test parameters

Parameter	Value
Load, N	25, 50, 75, 100, 125
Sliding speed, rpm	200, 400, 600, 800, 1000
Sliding speed, m/s	20.9, 41.9, 62.8, 83.8, 104.8
Track diameter, mm	40, 60, 80, 100, 120
Sliding distance, m	10.000

The design of the experiment was carried out using the Taguchi method for the test parameters given in Table 1. Accordingly, the experiment was carried out. The present set of experimental tests is conducted as per the Taguchi L25 orthogonal design array to identify the most significant variables by ranking them concerning their relative impact on the brake friction behaviour. There are three types of quality characteristics in the analysis of the signal-to-noise ratio, i.e. smaller-is-better, nominal-is-best and larger-is-better. Since the requirement is to maximise the brake performance through the selection of a proper parameter, a smaller-is-better quality characteristic is employed.

3.1 Preparation of sample pin and disc

The friction of commercially used cast iron brake rotor and their counterface made of commercial automobile non-asbestos brake pad material were studied in this investigation. The cast iron disc was cast with expendable mould in dry sand casting with the dimension 162 × 8 mm as shown in Figure 2.

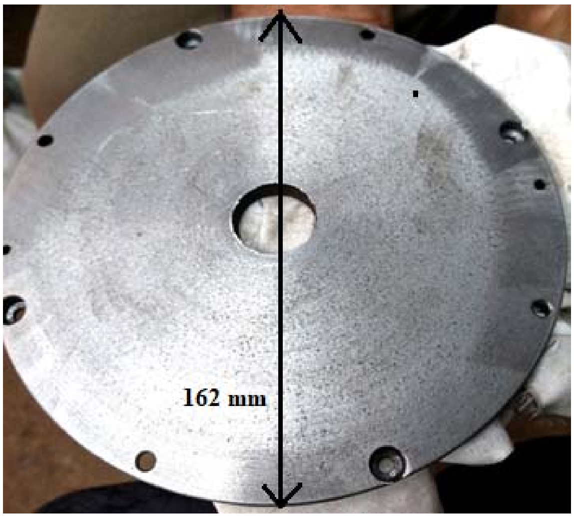

Figure 2. Cast iron disc used for test

The surface of the disc was machined to an average roughness value of *Ra* = 1.3 µm, measured with the help of the Handysurf E-35B instrument, which is the same as the roughness value of the sliding surface of the actual commercial brake rotor.

The brake liner sample is used for specimen preparation as shown in Figure 3. The friction materials of the brake pad were attached to a mild steel pin with the help of a strong epoxy adhesive. To ensure strong attachment between the liner and mild steel, small blind holes were drilled on the flat surface of a mild steel pin, a layer of epoxy adhesive was applied on both, the liner and mild steel pin, and kept them in pressure contact for 5 minutes. The forming process was used to make a hemisphere shape at the end of the pin as per the ASTM G99 test. The pin head is purposely made flat to depict the actual surface conditions of the brake pad.

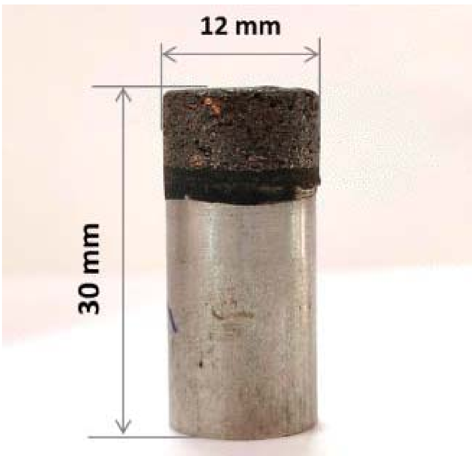

Figure 3. Specimen dimensions

3.2 Machine learning approach for prediction of tribological properties

As many previous investigations have found that constant speed and load in pin-on-disc tests are not sufficiently simulative of real brakes which experience a more dynamic environment. An artificial neural network (ANN) model was used to predict the properties of the brake system for the operating conditions based on the proposed mathematical model. The ANN model is trained by using the experimental data set of the pin-on-disc model.

The pin-on-disc dataset was imported using the "pandas" library and split into training and test sets using sklearn.model_selection. The "pandas" is a popular open-source library for data manipulation and analysis in Python. It provides a powerful data structure called DataFrame, which is similar to a spreadsheet in Excel, and allows one to work with tabular data in a more convenient and intuitive way. A DecisionTreeRegressor model was used to predict the friction force and coefficient of friction (COF). The model was trained on the training data and evaluated on the test data. Figure 4 shows the architecture of the DecisionTreeRegressor model.

In this diagram, each node represents a decision point in the model, where the input features are evaluated and the model decides which branch to follow based on the conditions. The leaves of the tree represent the predicted values for the target variables (friction force and COF).

The DecisionTreeRegressor model is a type of supervised machine learning algorithm that is commonly used for regression tasks. In this case, the goal was to predict the properties of a pin-on-disc system based on the operating conditions. The DecisionTreeRegressor model is useful for this task because it can handle both numerical and categorical data and can capture complex relationships between the input features and the output variables. Additionally, decision trees are interpretable models, meaning that we can understand how the model arrived at its predictions by examining the tree structure [21]. Overall, the DecisionTreeRegressor model was a suitable choice for this particular task.

To obtain values from the DecisionTreeRegressor model, we first split the dataset into training and testing sets using the train_test_split function from sklearn.model_selection module. The training set is used to train the decision tree model using the fit method of the DecisionTreeRegressor class. Once the model is trained, we use the test set to evaluate its performance by predicting the values of the target variables using the prediction method of the model.

A regression analysis was performed with the goal to find the best linear relationship between the dependent variable and the independent variable(s). This relationship is modelled using a linear equation, which can be used to make predictions about the dependent variable based on the values of the independent variables. The results show that both the pressure prediction and COF prediction models have very low MSE values (0.00), indicating that they are accurately predicting the target variables. Additionally, the R-squared scores are relatively high (0.93 and 0.97 for pressure and COF prediction, respectively), indicating that the models are explaining a large proportion of the variance in the target variables.

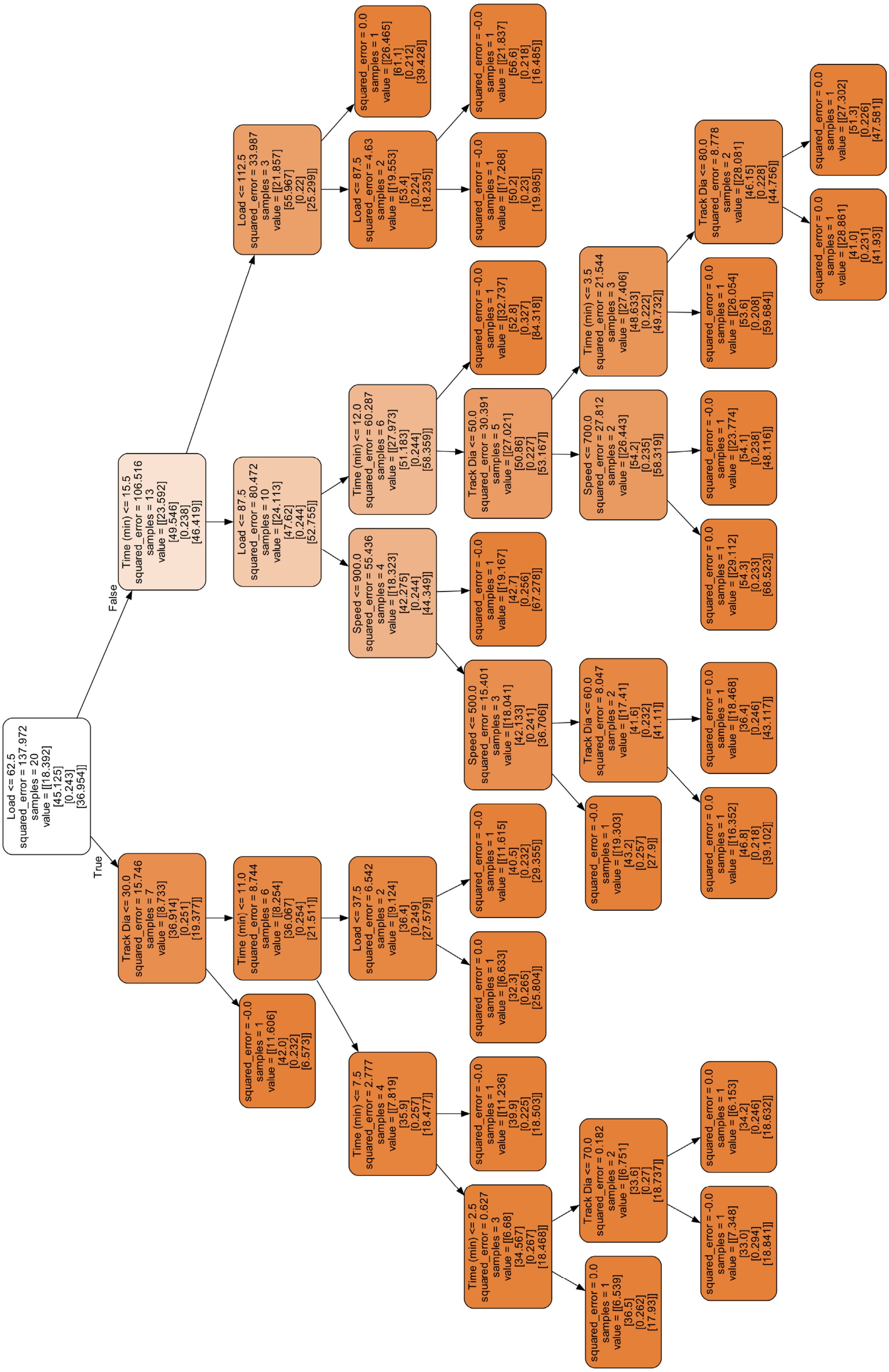

Figure 4. DecisionTreeRegressor model

Figure 5 is a scatter plot of the predicted values of the coefficient of friction (COF) versus the actual COF values, based on the linear regression model that was trained on the friction test data. The *x*-axis shows the actual COF values, while the *y*-axis shows the predicted COF values. Each dot on the plot represents a data point from the test data, and the colour of the dot indicates the pressure value associated with that data point. The closer the dots are, the better the model's predictions are.

Overall, the plot suggests that the model does a reasonable job of predicting the COF values, as most of the dots are clustered relatively close to the diagonal.

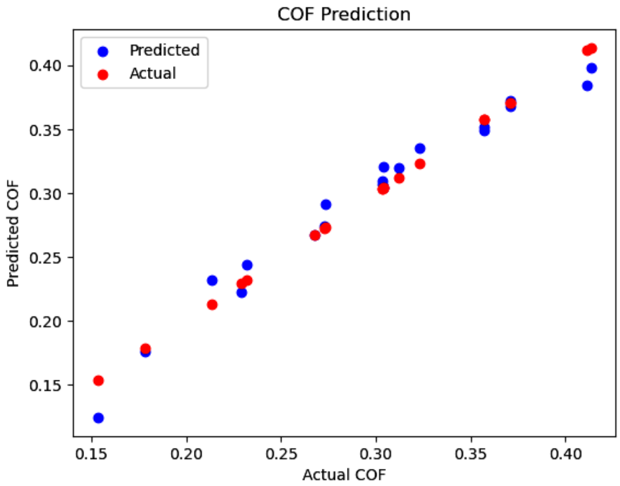

Figure 5. Actual vs. predicted COF

Table 2. Analytical database with the estimation of COF

Sliding speed, rpm	Load, N	Clamping force, N	Radius of asperity, µm	Calculated pressure, N/mm^2	Area, mm^2	Friction force, N	COF
200	25	7.280	0.1775	0.186	0.099	18.369	0.20
	50	11.620	0.1642	0.318	0.085	26.904	0.22
	75	17.268	0.1541	0.420	0.075	31.306	0.28
	100	32.730	0.1521	0.545	0.073	39.617	0.41
	125	27.300	0.1531	0.690	0.074	50.836	0.27
400	25	6.630	0.1652	0.161	0.086	13.783	0.24
	50	11.230	0.1540	0.279	0.075	20.817	0.27
	75	19.300	0.1545	0.422	0.075	31.633	0.31
	100	30.280	0.1540	0.559	0.075	41.634	0.36
	125	26.460	0.1492	0.656	0.070	45.851	0.29
600	25	7.340	0.1626	0.156	0.083	12.949	0.28
	50	12.040	0.1542	0.280	0.075	20.925	0.29
	75	19.300	0.1658	0.486	0.086	41.953	0.23
	100	30.287	0.1662	0.651	0.087	56.479	0.27
	125	26.465	0.1624	0.777	0.083	64.360	0.21
800	25	6.153	0.1624	0.155	0.083	12.860	0.24
	50	12.043	0.1625	0.311	0.083	25.807	0.23
	75	18.460	0.1595	0.450	0.080	35.931	0.26
	100	23.774	0.1585	0.592	0.079	46.717	0.25
	125	28.860	0.1602	0.756	0.081	60.943	0.24
1000	25	6.530	0.1660	0.162	0.087	14.052	0.23
	50	11.600	0.1623	0.310	0.083	25.681	0.23
	75	19.167	0.1595	0.450	0.080	35.931	0.27
	100	25.184	0.1695	0.677	0.090	61.100	0.21
	125	26.054	0.1585	0.740	0.079	58.397	0.22

4. Results and discussion

The database of estimated COF obtained from the analytical results of the equations and experimentation are discussed below, depicting a comparison between the COF obtained for increasing and decreasing levels of contact, actuating and friction forces.

Table 2 shows the estimated values of the calculated COF according to previously derived equations for different values of speeds and loads as mentioned in Table 1. The pressure presented in Table 2 is calculated according to Equation (1) and the area is calculated according to Equation (2). Clamping force is the force calculated through the pressure exerted by the brake caliper against friction force. It can be observed that the highest value of COF is 0.4 corresponding to 200 rpm and 100 N load and the lowest value is 0.2 for 25 N load with 200 rpm. As the speed is low the time of contact increases, thus increasing the area of contact with the elevated load. It is worth noting that, as speed increases the value of COF is stabilised around 0.2 – 0.3, which is expected from a brake friction material.

Following Table 3 represents experimental data obtained from pin-on-disc. Friction force and COF have been obtained directly through Winducom 2010 software. The time recorded is the required span for completing the set sliding distance, i.e. 1000 m. It is observed that, at 200 rpm and 100 N, the value of COF is 0.33 which is the highest and a similar observation is noted earlier from analytical data. The range of the COF is between 0.200 and 0.327. The lowest value of COF is 0.208 which is for the highest speed and load considered, i.e. for 1000 rpm and 125 N. This is due to the minimum time for interface and increased speed so the area of contact is reduced, therefore affecting the COF.

Table 3. Experimental data of pin-on-disc for COF

Sliding speed, rpm	Load, N	Track diameter, mm	Sliding speed, m/s	Time, min	Friction force, N	COF
200	25	20	20.94	53	7.2857	0.29
	50	40	20.94	27	11.6148	0.23
	75	60	20.94	18	17.2683	0.23
	100	80	20.94	13	32.7374	0.33
	125	100	20.94	11	27.3019	0.23
400	25	40	41.89	13	6.6327	0.27
	50	60	41.89	9	11.2363	0.22
	75	80	41.89	7	19.3027	0.26
	100	100	41.89	5	30.2870	0.30
	125	20	41.89	27	26.4654	0.21
600	25	60	62.83	6	7.3484	0.29
	50	80	62.83	4	12.0438	0.24
	75	100	62.83	4	18.4679	0.25
	100	20	62.83	18	21.8368	0.22
	125	40	62.83	9	29.1118	0.23
800	25	80	83.78	3	6.1534	0.25
	50	100	83.78	3	13.6967	0.27
	75	20	83.78	13	16.3524	0.22
	100	40	83.78	7	23.7741	0.24
	125	60	83.78	4	28.8610	0.23
1000	25	100	104.72	2	6.5387	0.26
	50	20	104.72	11	11.6055	0.23
	75	40	104.72	5	19.1671	0.26
	100	60	104.72	4	25.1847	0.25
	125	80	104.72	3	26.0543	0.21

Figure 6 represents the comparison between values of COF obtained through the theoretical and experimental investigation for variable speeds. The relationship between COF and load exhibits varying trends at different speeds, rather than being consistently proportional or showing uniform increases. The difference between the theoretical and experimental values of the COF is mainly due to the varying area of contact, as discussed earlier. Also, due to wear, the area of contact changes; hence, the COF range seems to deviate. At higher speeds, the calculated area for the theoretical method appears to be increased as the considered pressure is high. In the experimental data, not much difference in values of COF is seen even for high.

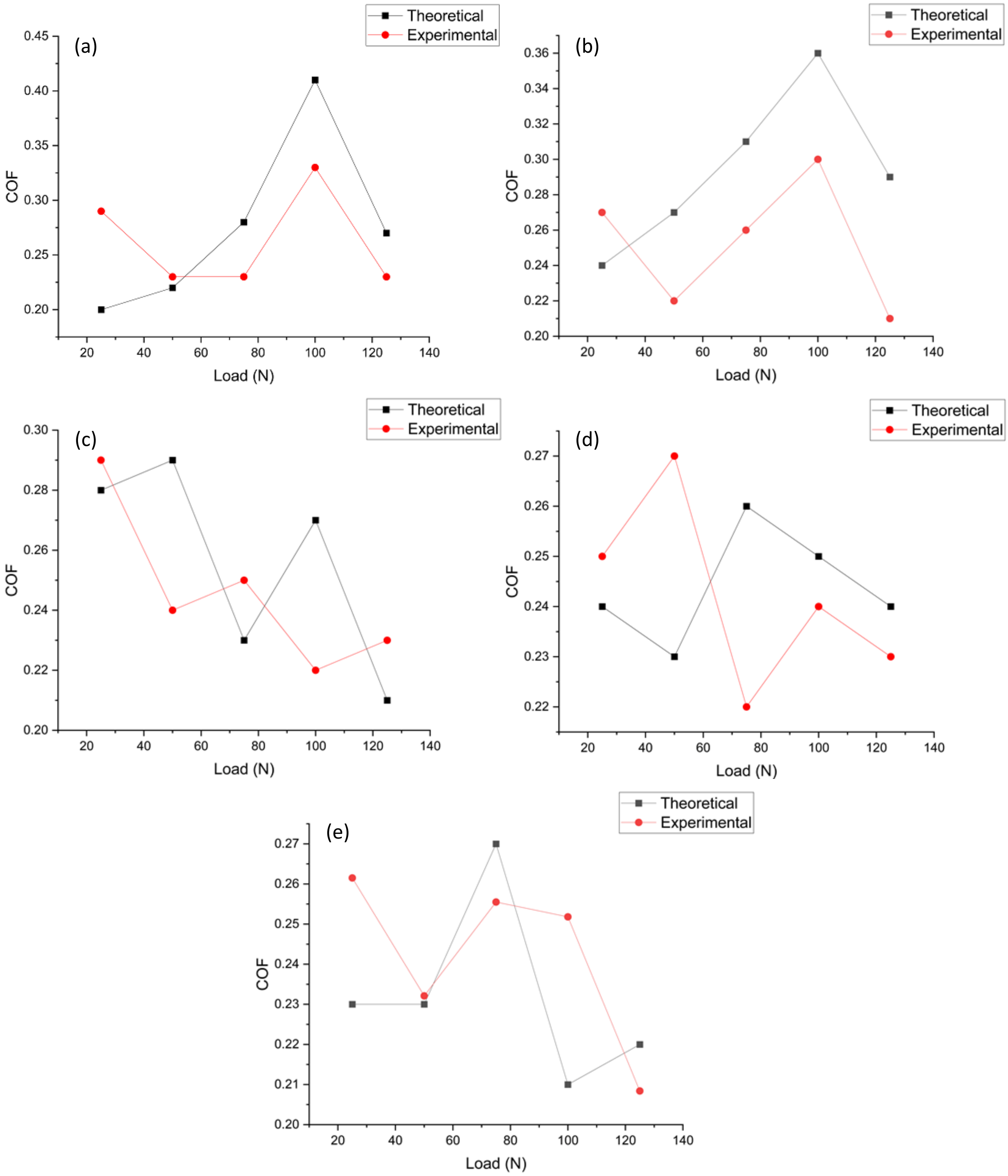

Figure 6. Comparison between theoretical and experimental values of COF at: (a) 200 rpm, (b) 400 rpm, (c) 600 rpm, (d) 800 rpm and (e) 1000 rpm

The range of COF for the experimental method is between 0.21 and 0.33 and the range obtained from the theoretical equation is from 0.20 – 0.41. As the surface area during the initial stage of braking is the same, the initial values of COF for all the speeds are similar and in the range between 0.20 and 0.25. This changes gradually with a change in the contact pressure. This is also occurring due to coupling load, i.e. the external force applied to a system or material that influences the formation of micro-junctions and affects the adherence between contacting surfaces.

In Figures 6a and 6b, the value of COF after 50 N can be seen increasing drastically for the experimental method. Whereas, for the theoretical method, the value of COF increases moderately as only the points of contact at the interface are considered. This is the main perk of contact mechanics, which aids in visualising and evaluating the actual contact area in the tribological interface. For higher speeds, i.e. for 800 and 1000 rpm, the value of COF is seen to decrease linearly due to less contact time and increased pressure.

5. Conclusions

A mathematical model is proposed in the present work for calculating the actual COF. It is derived by using basic concepts of contact mechanics and material properties. An experimental evaluation was carried out on the pins made of brake pad friction material of passenger vehicles, under various speeds, pressures and loading conditions on pin-on-disc. Finally, the experimental and analytical results were compared for validation.

The analytical evaluation and experimental evaluation show a similar trend for COF. The value of COF obtained through the proposed method has been in the range of 0.2 – 0.4, which is the same for both evaluations and falls in the normal range as per the reported literature.

There is no fundamental difference between the experimental and theoretical values. The obtained experimental values greater than the theoretical ones are explained as follows: (a) the increase of the real contact area due to the friction force, and (b) the occurrence of additional forces when the material deforms.

The obtained experimental values lower than the theoretical values are due to the different adherence of the materials during the formation of micro-junctions. In the theoretical model, we took into account: (a) the mode of surface processing, and also (b) the characteristics of the material, but we have not taken into account the coupling load.

The analytical values are consistent with experimental observations and can be used to estimate the COF for developing an efficient brake system using different brake pad materials, considering the surface properties of the friction material and operating conditions of the brake system.

The ANN model is useful in predicting the values of the COF and interface pressure for brake disc/brake pad systems with a regression coefficient of 0.97, which can be further used for determining the tribological properties of new friction materials and their compatibility for efficient brake systems.

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