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Numerical study of a centred crack on an elastoplastic material by the FEM method

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Abstract

The numerical study of crack propagation is a very complex and important problem for knowing the lifetime of a structure. Nowadays, the modelling of crack propagation by various numerical methods plays a beneficial part in solving problems in fracture mechanics. Significantly used in biomechanics, biomaterials and structural calculation, it also makes it possible to deal with certain problems of fatigue of materials. In this article, a numerical study based on the finite element method (FEM) was used for a two-dimensional model, of an elastoplastic material, containing a central crack. The article is also based on the study of different cracking factors, such as stress intensity factors K_{I} and K_{II}, J-integral and strain energy. On the other hand, the contrast of these parameters was precisely centred on the five contours of the crack front. In addition, Abaqus computer code was used to obtain different results, moreover, CPS4R elements were used for modelling.

1. Introduction

The numerical study is a theory widely used by engineers that allows the representation of one of the important physical phenomena in fatigue crack propagation. Originally, numerical studies were developed for isotropic homogeneous materials. However, it is commonly used to characterise the different singularity parameters based on the evolution of the stress intensity factors in different modes, the contour integral (J-integral) and the strain energy. On the other hand, Laribou and Qotni [1] have examined and verified the analytical calculations of the stress intensity factor (SIF) through an empirical approach of the form factor in mode I, by the finite element method with the software Abaqus in the linear elastic domain for two different cracks. The first has a circular section shape, and the second has an elliptical section shape, and both shapes contain a central crack under a uniform tensile load. El Fakkoussi et al. [2] calculated the stress intensity factor K_{I}, in mode I, by the finite element method (FEM) and the extended finite element method (XFEM) in the linear elastic domain, of a longitudinal semi-elliptical crack of a tube. Yu and Kuna [3] presented the interaction integral (I-integral) method to extract the individual stress intensity factor (SIF) and stress T of a crack in single materials or at a bi-material interface. On the other hand, Lal et al. [4] presented a study based on the effect of the length and crack angle on the mixed-mode stress intensity factor (MSIF) of a functionally graded material plate cracked at the centre under biaxial stress. Khatri and Lal [5] presented the stochastic fracture analysis of an isotropic plate with a hole and emanating crack under biaxial loadings by the XFEM method. In addition, de Araújo et al. [6] presented an adaptive methodology for the finite element analysis of an elastoplastic material with a two-dimensional cracked structure. Alshoaibi and Fageehi [7] proposed a study based on the formulation of the finite element method, to analyse the problems of fatigue crack propagation, according to the linear elastic fracture mechanics (LEFM) theory. Goud et al. [8] showed the variation of the stress concentration factor for different geometries of a crack in a thin plate. Deng et al. [9] proposed a numerical analysis of crack propagation on a sample under stress, and the Paris law was used to predict fatigue life. On the other hand, a new method stretching finite element method (SFEM) has been used by Bentahar et al. [10] and Bentahar and Benzaama [11] to characterise the stress intensity factors of an initial crack.

This article contributes to the knowledge of the properties of the central crack of an elastoplastic material, by studying and diagnosing the crack tip zone.

2. Numerical modelling

The stress intensity factor K_I is an essential parameter in fracture mechanics, which allows us to know the state of stress and strain at any crack point Saverio [12]. According to Ewalds and Wanhill [13] the stress intensity factor is given by the following relationship:

$$K_I = F\sigma\sqrt{a\pi}, \tag{1}$$

where σ is the applied stress, a is the crack length and F is the geometric correction factor of the used model:

$$F = 1.12 - 0.23\left(\frac{a}{w}\right) + 10.6\left(\frac{a}{w}\right)^2 - -21.7\left(\frac{a}{w}\right)^3 + 30.4\left(\frac{a}{w}\right)^4, \tag{2}$$

where w is the length of the plate.

The stress intensity factor K_{II} is calculated by the relation:

$$K_I \sin\theta + K_{II}(3\cos\theta - 1) = 0, \tag{3}$$

where θ is the kinking angle during crack propagation.

2.1 Maximum circumferential stress criterion (MCSC)

The maximum circumferential stress criterion (MCSC) was introduced by Erdogan and Sih [14] for elastic materials. It specifies that the crack propagates in the direction for which the circumferential constraint is maximum:

$$\mathrm{tg}\frac{\theta}{2} = \frac{1}{4}\frac{K_I}{K_{II}} \pm \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}, \tag{4}$$

where K_I and K_{II} are the stress intensity factors corresponding to cracking mode I and II, respectively.

2.2 Different cracking modes

The presence of defects may be in the form of internal cracks or surface cracks. Fracture mechanics analysis correlates parameters from loading, geometry and material. Then, it predicts the conditions under which a crack can propagate and possibly lead to the complete failure of the structure.

In the continuous medium, cracking is an irreversible separation phenomenon. Three main modes of cracking can be distinguished (Fig. 1).

- aperture mode (mode I),
- shear mode (mode II) and
- anti-plane shear mode (mode III).

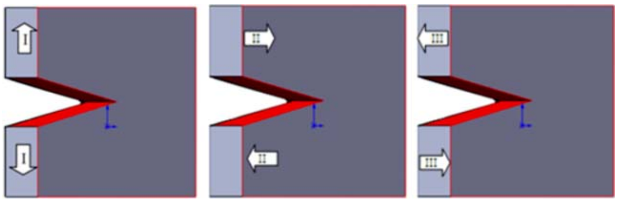

Figure 1. Illustration of fracture cracking modes

2.3 Stress fields and modelling elements

Tada et al. [15] gave the general 2D stress field equation near the crack front, to define the stress intensity factor. The CPS4R elements were used around the crack front. Further, Shi et al. [16] proposed a plastic history field to perform the energy decomposition at the crack front. Figure 2 shows the stress field near a crack point.

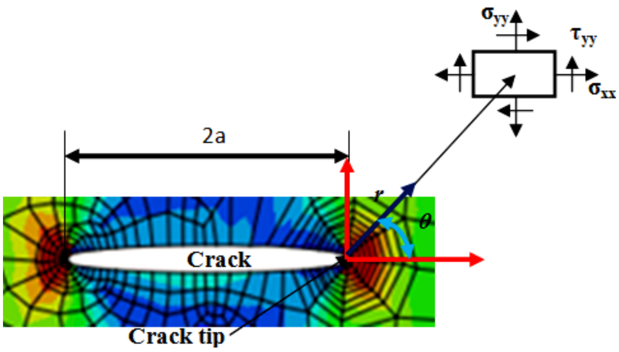

Figure 2. The stress state at the level crack front

The stress field in 2D near the crack front defied by the stress intensity factor is given by Equation (5) [15]:

$$\sigma_{i,j}^{I,II}(r,\theta) = \frac{K_{I,II}}{\sqrt{2\pi r}} f_{ij}(\theta), \tag{5}$$

where $\sigma_{i,j}^{I,II}$ is the stress field associated with mode I and $K_{I,II}$ is SIF in modes I and II.

In addition, Equation (6) illustrates the constraints on the two axes (x and y).

$$\sigma_{xx} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{yy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}. \tag{6}$$

2.4 J-integral

Several fracture mechanics authors have made it possible to model the problem of crack propagation in depth and have developed calculation methods. Among these authors Rice [17] and Bui [18] with contour J-integral, and Nguyen [19] and Destuynder [20] by introducing an arbitrary field in the formulation of the integral they have approached. Figure 3 illustrates the five contours of the crack tip.

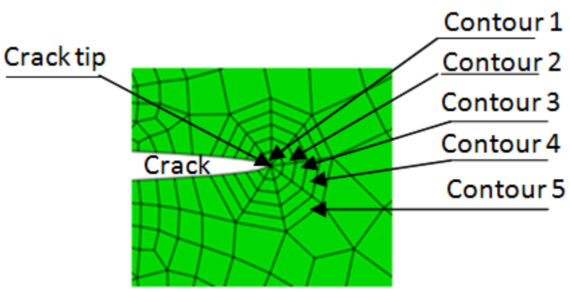

Figure 3. Definition of the contours for the evaluation of the J-integral

3. Material properties and studied model

Table 1 presents the mechanical and physical properties of the elastoplastic material (aluminium alloy), for which the plastic properties vary between 80 and 120 MPa for points 1 and 2 (Fig. 4).

Table 1. Mechanical and physical properties of the material

	Plastic		Elastic		Density, kg/m^3
	Yield stress, MPa	Plastic strain, %	Modulus of elasticity, GPa	Poisson's ratio	
1	80	0	70	0.33	7872
2	120	0.3			

The studied plate is a structure of the rectangular form, with dimensions ($R \times L$) 10 × 20 mm, made up of an elastoplastic material whose mechanical properties are presented in Table 1. The centred crack length is $2a$ = 2 mm, and the boundary conditions are shown in Figure 4. The embedding is applied to surface 1, and the tensile stress has been applied to surface 2. The mesh is made up of the CPS4R elements as shown in Figure 5.

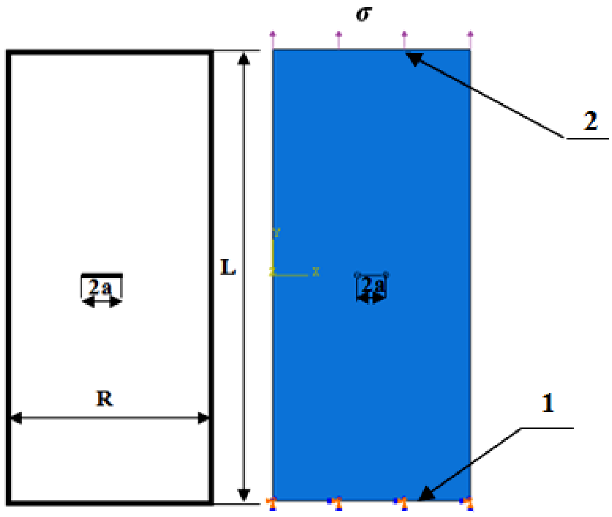

Figure 4. Studied model: (a) dimensions and (b) boundary conditions

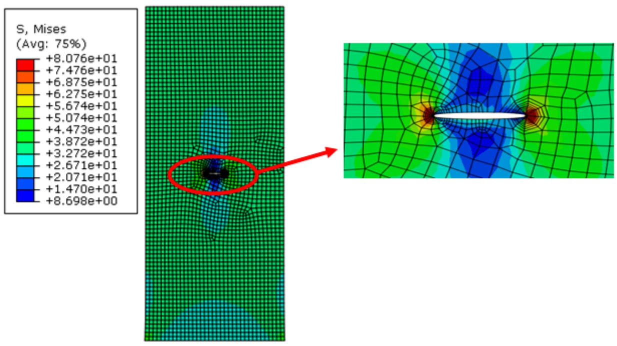

Figure 5. FEM model with the construction mesh

4. Results and discussion

The results part is mainly based on the study of different characterisation parameters, such as the stress intensity factors K_{I} and K_{II}, the J-integral and the stress energy.

Figure 6. shows the variation of the stress intensity factor K_{I} as a function of time concerning the model which is studied. The study is based on the state of variation of K_{I} at the level of the crack front of the zone of singularity. The state of the stress intensity factor K_{I} variation is compared between the different contours. Thus, the optimisation of the time interval t is confined between $0.25 \leq t \leq 0.69$ s. This comparison shows a good proportionality between the five contours, concerning the results obtained.

Figure 7 shows the variation of K_{II} as a function of time. It can be said that the results obtained from the stress intensity factor K_{II} concerning the different contours are proportional to each other.

In addition, we can notice that the results are satisfactory, notably in the last three contours. This comparison is made in the case of t = 0.25, 0.3, 0.4, 0.5, 0.6 and 0.69 s. Indeed, the more the time increases, the more the K_{II} decreases.

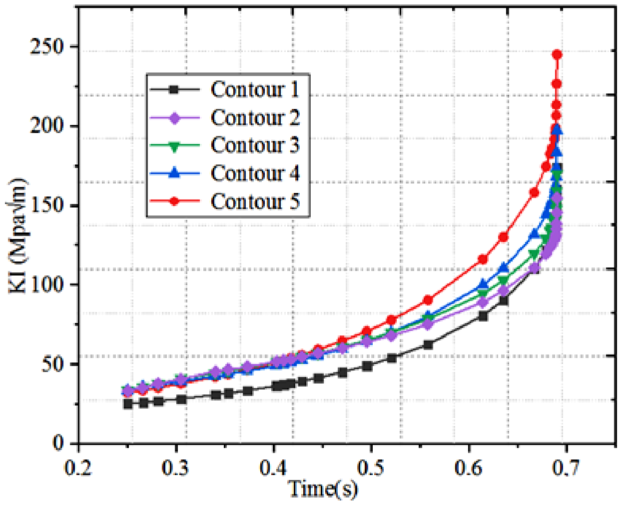

Figure 6. Variation of stress intensity factor K_I

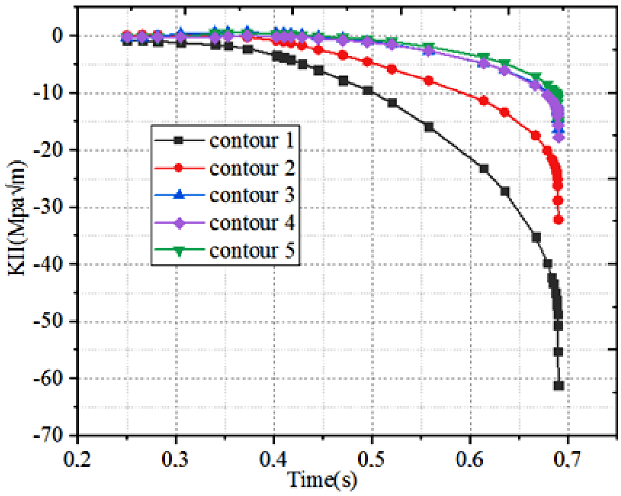

Figure 7. Variation of stress intensity factor K_{II}

Figure 8 shows the variation of J-integral in the different periods, i.e. from t = 0.25 s up to t = 0.69 s, by the FEM method. Note that crack propagation causes an increase in the J-integral. Indeed, from this figure, we can notice that the values of J-integral are almost constant up to t = 0.45 s.

Figure 9 presents the variation of strain energy as a function of time. It can be seen that values obtained for the strain energy in the time interval from t = 0.25 s to t = 0.69 s are proportional to each other.

The strain energy remains stable until t = 0.3 s after which the strain energy starts to increase for the contours 1 and 2, while for the other contours, the energy remains slightly constant. However, after the value of t = 0.4 s, the strain energy increases rapidly for all contours. The strain energy for the first two contours varies in the interval from 0 to 35 J, for the time interval from 0.25 to 0.69 s. The study of the energy at the level of the crack front has been done by Bentahar et al. [21] in the case of the strain energy, by the XFEM method; by Bentahar [22] in the case of the analysis of the energy dissipation; and by Bentahar [23] in the case of fatigue analysis of an inclined crack propagation.

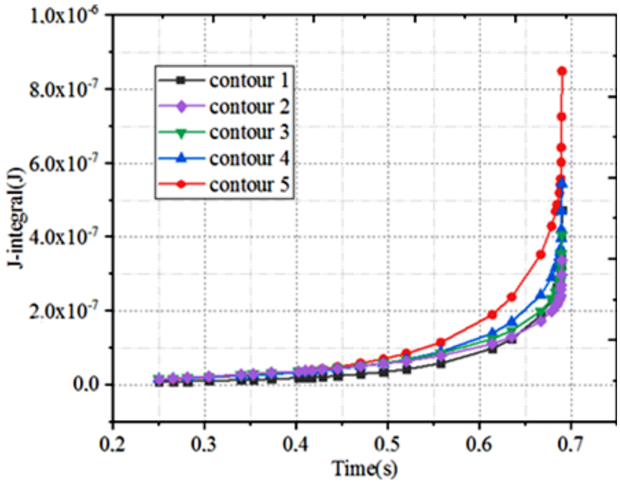

Figure 8. Variation of J-integral

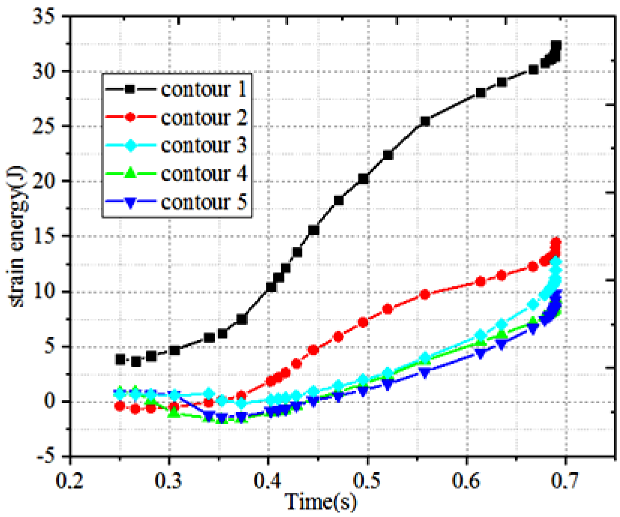

Figure 9. Variation of strain energy

5. Conclusion

The object of this study was the analysis of the propagation of centred crack on a structure of an elastoplastic material. In addition, linear fracture mechanics tools were used, such as the crack propagation criterion, the stress state at the crack front and the mesh.

The variation of the different crack parameters was studied, such as the stress intensity factors K_I and K_{II}, the contour J-integral and the strain energy.

The finite element method (FEM) was chosen for the simulation. The different parameters have been studied as a function of time in the interval from 0.25 up to 0.69 s. All the obtained results are proportional with respect to the different contours of the crack front.

The optimisation of the time interval is confined between 0.25 and 0.69 s. The more the time increases, the more the K_{II} decreases. It can be seen that the values obtained for the strain energy are proportional to each other. The J-integral variance has a nearly constant value up to 0.45 s.

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