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Influence of lubricant type on churning power loss of worm gear

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Abstract
This paper presents the results of experimental tests of the churning power loss of a single-stage worm gear. The churning power loss is a load-independent loss that occurs when the gear churns through the lubricant. This study will solely look at how churning power losses are affected by mineral and synthetic oil on a test worm gear pair where the worm shaft is made of 16MnCr5 and the worm wheel is made of CuSn12. As a lubricant, mineral and synthetic oil were used for gears operating at variable speeds, different oil volumes and positions of the worm shaft. The direct torque measurement method was used in a thorough experimental investigation. A new test rig was devised, built and put into service to quantify churning power loss in unloaded situations. According to the experiment results, churning power loss values for mineral oil are 20 – 25 % greater than those of synthetic oil, which has almost the same kinematic viscosity at a temperature of 40 °C.

1. Introduction

The churning power loss, windage power loss and oil seal losses are part of load-independent power losses [1]. When gears drag in splash oil, they experience churning power losses; when they drag in the air, they experience windage power losses [2-4]. The main function of lubrication is to avoid the metal to metal contact and disseminate the heat produced by gears [5]. Muminović et al. [6] experimented on worm gearbox with two types of lubricating oil, synthetic and mineral and concluded that synthetic oils provide a higher level of efficiency compared to mineral oils. Mineral oils have a higher coefficient of friction compared to synthetic oils in the same operating conditions. Both, mineral and synthetic base oils are typically used with industrial worm gears [6,7].

Since most of these machines are splash lubricated, they need to run at the right level of lubricant in order to raise and splatter lubrication across every surface within the machine. This is the point at which lubricant churning may cause problems. The machine needs to work harder to push through the extra lubrication if the lubricant level is too high, either because too much oil has been injected or because the bearings have been over-greased [8-10]. Daily and Nece [11] experimented on enclosures and bladed discs. Terekhov [12] and Boness [13] looked into the churning power losses in dip-lubricated gear transmissions under various operating circumstances and developed empirical formulas. Using the direct torque measurement technique, various types of research have been conducted to develop empirical connections to investigate churning losses. [13-15]. Luke and Olver [9] found certain discrepancies between the results of Terekhov and Boness. Michaelis and Höhn [15] experimented on cylindrical gears and discovered that the churning power loss depends mainly on the viscosity of the lubricant and not on the type of lubricant. Changenet and Velex [16] examined the influence of housing dimensions on churning loss. Kolekar et al. [3] investigated the churning power loss of a single gear by using the inertia rundown technique. Skulić et al. [17] examined the influence of worm gear material and lubricant on the efficiency and coefficient of friction, and also concluded that higher values of the efficiency of the worm drive 42CrMo4/CuSn12 cause lower values of the coefficient of friction by 14 to 21 %, as well as less wear of the worm gears. In recent years several numerical methods have been developed to investigate the churning power loss for parallel axis gear [18-20]. However, the experiment readings were used to develop the mathematical model for the churning power loss of worm gear [18]. Liu et al. [21] investigated the churning loss and lubrication performance of the gearbox with a guide plate based on the MPS algorithm.

The literature survey uncovers that most of the experimental studies on parallel axes of gear and most of the experimental studies devoted to examining churning power losses were constrained to a disc or a single gear. However, our previous paper [1] briefly described the effect of worm speed, immersion depth, worm shaft position, lubricant temperature and lubricant type on the churning power loss of worm gear. The present research exclusively focuses on a single parameter, the effect of lubricant type on churning power loss. So, it can be helpful to select the proper lubricant to avoid the churning power of worm gear. The impact of mineral-based and synthetic-based oil on the single-stage worm gear's churning power loss is examined by using a developed test rig.

2. Materials and methods

The experimental studies were performed on a specially designed torque measurement test rig which work is based on the direct torque measurement technique. The schematic representation of this developed test rig is given in Figure 1. Through a torque sensor and a flexible coupling, an electric motor powers the worm gear. The torque sensor's vibration is lessened by the placement of the foot-mounted bearings on both sides. The rotational torque sensor provides a direct assessment of the resisting torque with an accuracy of ± 0.01 Nm. A variable frequency drive is utilised to measure and control the worm's speed.

The oil temperature, air pressure inside the gearbox and oil level can be measured with the help of different facilities available on the test rig. The gearbox's top was chosen for the placement of the non-return air valves in order to mitigate windage loss. The test gear pair is a single-stage worm gear pair as shown in Figure 2.

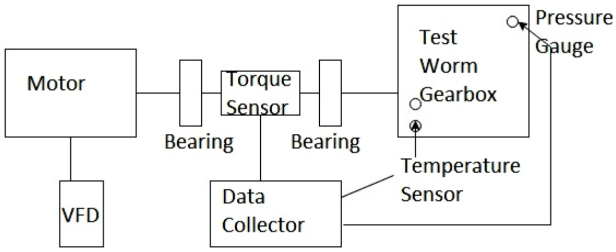

Figure 1. Schematic representation of the test rig

Figure 2. Single-stage worm gear pair

The size of the test gearbox is 180 × 180 × 280 mm. The other dimensions of the gear pair are given in Table 1.

Table 1. Worm gear pair geometric properties

Property	Worm shaft	Worm wheel
No. of teeth	single-start	30
Material	16MnCr5	CuSn12
Module, mm	3	
Pressure angle, °	20	
Centre distance, mm	75	
Outer diameter, mm	40	132
Reduction ratio	30 : 1	

The oil levels of the gearbox are shown in Figures 3 and 4. The gearbox was operated with two different orientations. Both side oil levels were not at the same level so the volume of lubricant was kept constant throughout the experiment as shown in Figures 3 and 4. The experiments are conducted at various rotational speeds of 1000, 1200 and 1400 rpm, with different oil volumes of 1.5, 2.1 and 2.7 litres, and for different lubricants (oil A and oil B). The friction of the gears raises the temperature of the lubricating oil. All the readings are taken at a lubricant temperature of 40 °C.

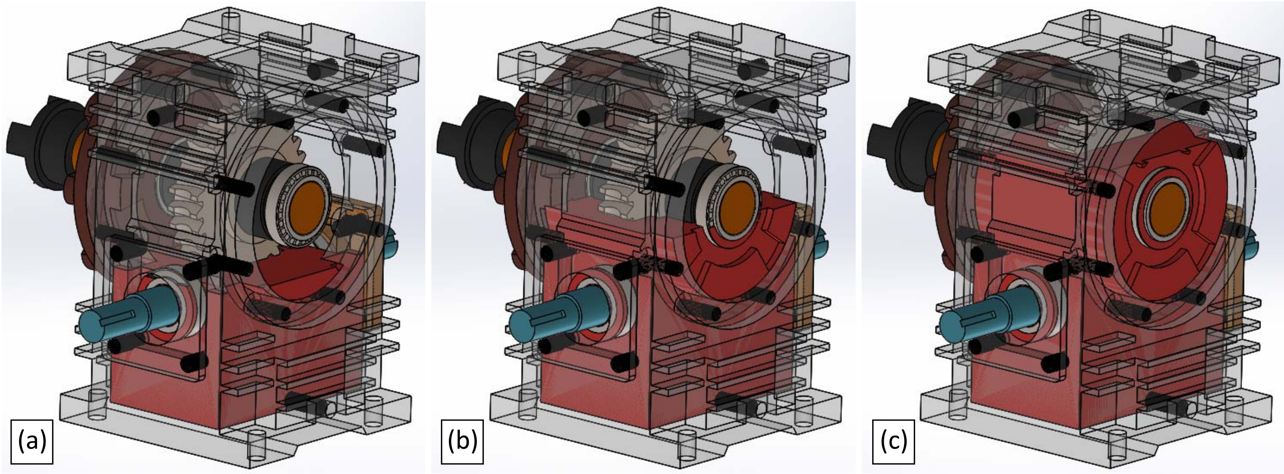

Figure 3. Oil levels when the worm shaft is at the bottom for the volume of: (a) 1.5 litres, (b) 2.1 litres and (c) 2.7 litres; reprinted from Chothani [22], copyrighted by the author

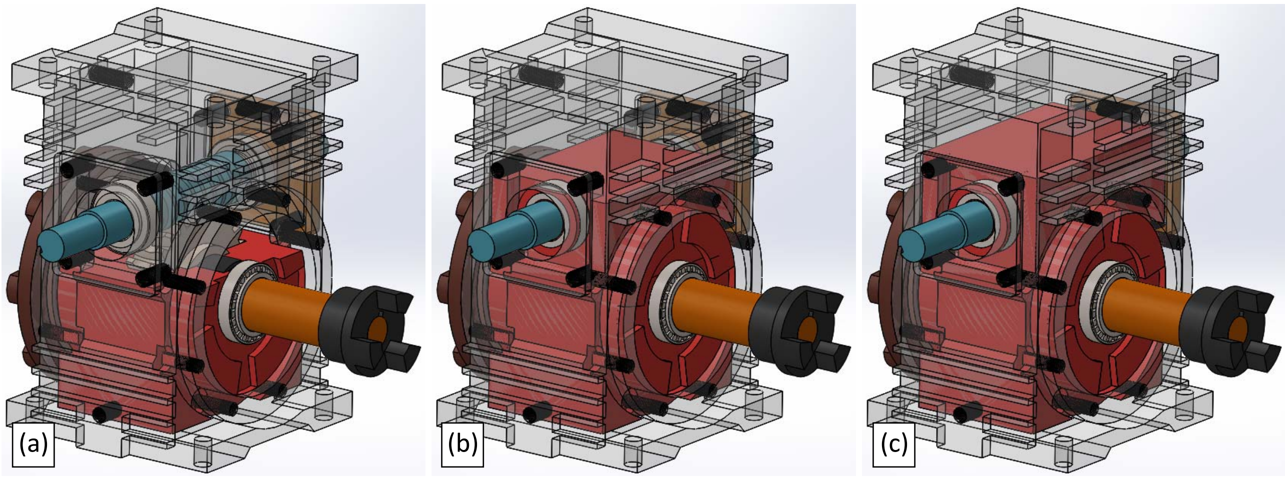

Figure 4. Oil levels when the worm shaft is at the top for the volume of: (a) 1.5 litres, (b) 2.1 litres and (c) 2.7 litres; reprinted from Chothani [22], copyrighted by the author

Figure 3 shows the lubricant level for different volumes when the worm shaft is at the bottom. The levels of lubricant are measured from the centre of the worm shaft. These are the 20, 60 and 115 mm for the 1.5, 2.1 and 2.7 litres, respectively. Figure 4 shows the lubricant level for different volumes when the worm shaft is at the top. The levels of lubricant are measured from the centre of the worm wheel. These are the 41, 95 and 113 mm for the 1.5, 2.1 and 2.7 litres, respectively.

The test matrix is formed based on pilot studies, industrial applications and test rig limitations. With the help of a torque sensor and a variable frequency motor, the torque and speed were monitored to compute the input power P_{immersed} [18] at a lubricant temperature of 40 °C. The gearbox oil temperature was allowed to drop to room temperature before taking another set of readings. The power loss $P_{\text{non-immersed}}$ was computed at the same speed as in the immersed scenario when the worm shaft and worm wheel were not submerged in oil [18]. For the non-immersed condition, a small quantity of oil was supplied to the bearing area and the gear work area. Considering that the estimations of friction power loss for both the immersed and non-immersed conditions were roughly equal [12], the difference in power loss between the two lubrication conditions was due to the gears churning [18].

In the experiments, the churning torque is calculated first by subtracting immersed and non-immersed conditions. After that, churning power loss P_c was calculated based on the value of churning torque T in Nm and the worm speed N in rpm as $P_c = 2\pi NT/60$.

Mineral and synthetic oils are commonly utilised in commercial worm gears. Two different gear oils, oil A and oil B, were used to investigate the impacts of different types of oils with nearly identical viscosity. Oil A was mineral oil (EP 140) and oil B was synthetic oil (PAO 320), as shown in Table 2. Mineral oils are widely used due to their good lubricating properties and cost-effectiveness. Extreme pressure (EP) mineral gear oils are used more extensively in worm gears. The EP (antiscuff) additive reacts with the metal surface to form a soft, slippery layer that prevents severe wear and welding under conditions of high pressure and temperature [23,24]. Polyalphaolefins (PAOs) and polyethylene glycols (PAGs) are the most common types of synthetic lubricants. These have a lower friction coefficient in a gearbox, better film strength and a better relationship between viscosity and temperature [23,24]. The worm gear can be lubricated in several ways, but splash lubrication was employed in this experiment to measure churning power loss.

Table 2. Properties of lubricants used in the experiment

Property	Oil A (mineral oil EP 140)	Oil B (synthetic oil PAO 320)
Density at 15 °C, kg/m^3	880	790
Kinematic viscosity at 40 °C, mm^2/s	312	330
Kinematic viscosity at 100 °C, mm^2/s	33	35.5
Viscosity index	95	162
Flash point, °C	220	208
Pour point, °C	– 9	– 39
Specific gravity	0.910	1.054

3. Results and discussion

The experiment's readings and calculations are given in Tables 3 and 4. Figures 5 and 6 depict the effect of lubricant type at different lubricant levels. Figure 5a shows that the churning power losses for oil A and oil B are significantly close for worm speeds up to 1200 rpm, although there is a little difference above this speed. At 1400 rpm, oil A had a nine percent higher churning power loss than oil B. It means that the churning power loss for mineral oil is nine percent more than that of synthetic oil at 1400 rpm for a smaller lubricant volume.

For a 2.1-litre oil volume (Fig. 5b), the difference in churning power loss for oil A and oil B is twenty-one percent, five percent and twenty-six percent at speeds of 1000, 1200 and 1400 rpm, respectively. Similarly, for an oil volume of 2.7 litres (Fig. 5c), the difference is 16, 6 and 27 % at speeds of 1000, 1200 and 1400 rpm, respectively. It demonstrates that at submerged settings, there is an early difference in churning power loss; as speed rises, this gap narrows to a modest speed; however, the churning power loss gap between mineral oil and synthetic oil rises with speed.

As per Figure 6a, a very little or ignorable churning power loss is obtained for 1.5 litres, as the lubricant is not touching through the worm shaft in this position as described in Figure 4a. It is found that at higher speeds and submerged conditions (volume higher than 1.5 litres), the churning power loss of mineral oil is higher compared to synthetic oil. Figure 6 shows the churning power loss gap between the mineral and synthetic oil when the worm is at the top position. These differences are very minor or negligible for the gearbox having a lubricant volume of up to 2.1 litres, as shown in Figure 6b. However, for the highest volume (volume of 2.7 litres) and high speeds, the type of lubricant influences the churning power loss of the tested gearbox, as shown in Figure 6c. The churning power loss for mineral oil at high speed is much higher than for synthetic oil. In general, the average churning power loss for oil A (mineral oil) is slightly higher than for oil B in all cases.

The primary function of the lubricant is to provide cooling and reduce heat generated due to the friction of the gear, as well as to increase efficiency [25]. Figure 7 depicts the churning power loss and heating rate for oil A and oil B at 1000 rpm, lubricant volume of 2.7 litres and worm in the bottom position. For this, experiments were continued for 40 minutes with different lubricants and temperature and churning power loss were measured every 5 minutes. The results of the experiments are given in Table 5.

The difference in churning power loss between oil A and oil B is initially large, but it decreases over time and is equal after 30 minutes, as shown in Figure 7a. Oil A has a much higher initial churning power loss than oil B as shown in Table 5. After 30 minutes of heating, the churning power loss for both oils is approximately equal. Similarly, the heating rate for both oils is the same as shown in Figure 7b.

For the worm at the top position, the churning power loss at 1000 rpm and lubricant volume of 2.7 litres, oil A is 10 to 20 % higher for the initial 20 minutes of the experiment. After this period the

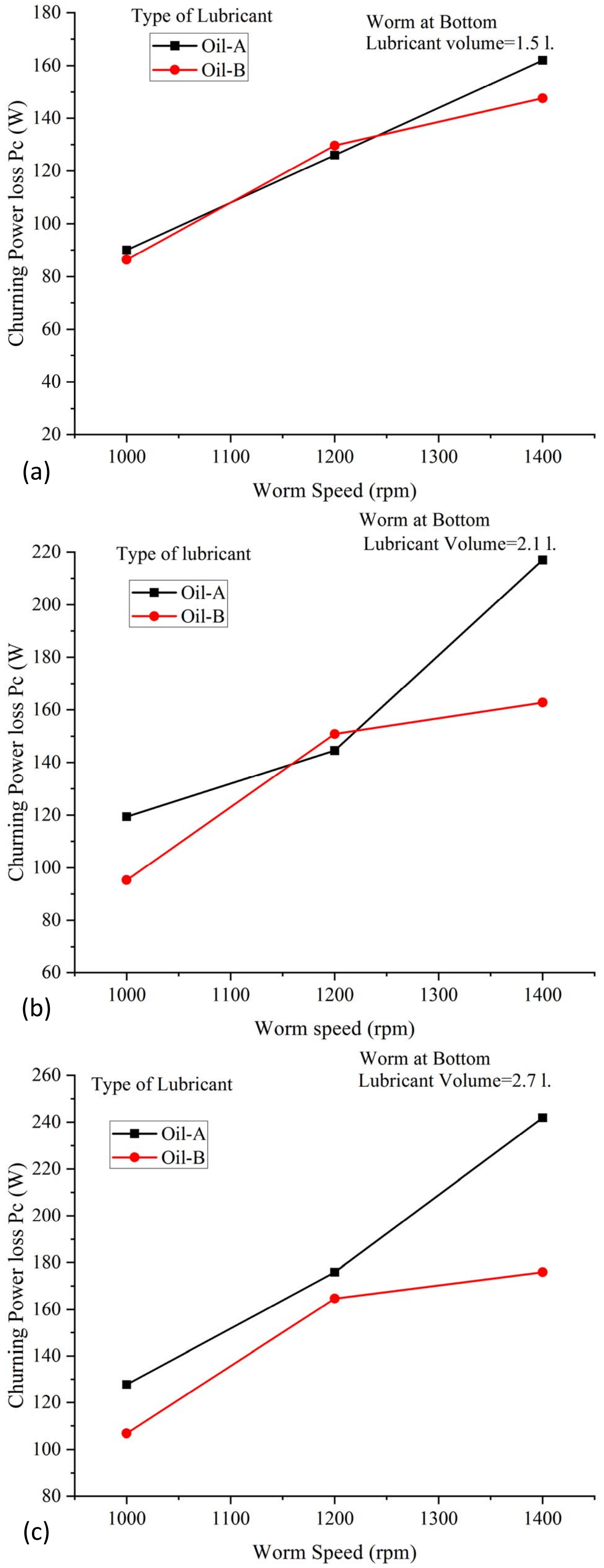

Figure 5. Impact of lubricant type on churning power loss when the worm is at the bottom position and the lubricant temperature is 40 °C for the volume of: (a) 1.5 litres, (b) 2.1 litres and (c) 2.7 litres

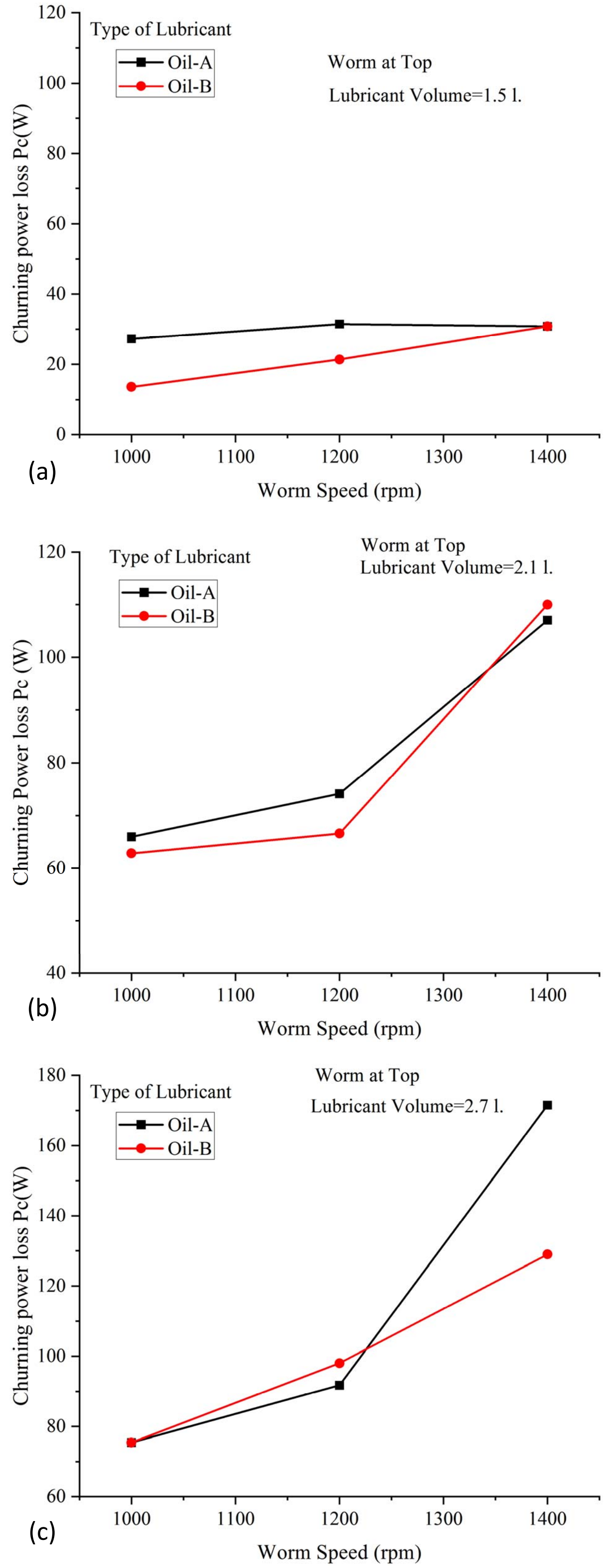

Figure 6. Impact of lubricant type on churning power loss when the worm is at the top position and the lubricant temperature is 40 °C for the volume of: (a) 1.5 litres, (b) 2.1 litres and (c) 2.7 litres

gap is reduced, as shown in Figure 8a and Table 6. The heating rates of lubricant due to friction are the same for both the lubricant, as shown in Figure 8b. It is found that there is a very minor influence on the type of lubricant on the heating rate of lubricant for the tested gearbox.

Table 3. Experiment readings and calculations when the worm is at the bottom position and the lubricant temperature is 40 °C; adapted from Chothani [22], copyrighted by the author

No.	Worm speed, rpm	Oil volume, l	Oil type	Input torque, Nm	Churning torque, Nm	Churning power loss, W
1	1000	1.5	A	2.29	0.84	87.96
2	1000	2.1	A	2.59	1.14	119.38
3	1000	2.7	A	2.67	1.22	127.76
4	1200	1.5	A	2.49	0.97	121.89
5	1200	2.1	A	2.67	1.15	144.51
6	1200	2.7	A	2.92	1.40	175.93
7	1400	1.5	A	2.72	1.05	153.94
8	1400	2.1	A	3.15	1.48	216.98
9	1400	2.7	A	3.32	1.65	241.90
10	1000	1.5	B	2.24	0.79	82.73
11	1000	2.1	B	2.36	0.91	95.29
12	1000	2.7	B	2.47	1.02	106.81
13	1200	1.5	B	2.53	1.01	126.92
14	1200	2.1	B	2.72	1.20	150.80
15	1200	2.7	B	2.83	1.31	164.62
16	1400	1.5	B	2.63	0.96	140.74
17	1400	2.1	B	2.78	1.11	162.73
18	1400	2.7	B	2.87	1.20	175.93

Table 4. Experiment readings and calculations when the worm is at the top position and the lubricant temperature is 40 °C; adapted from Chothani [22], copyrighted by the author

No.	Worm speed, rpm	Oil volume, l	Oil type	Input torque, Nm	Churning torque, Nm	Churning power loss, W
1	1000	1.5	A	1.83	0.26	27.23
2	1000	2.1	A	2.20	0.63	65.97
3	1000	2.7	A	2.29	0.72	75.40
4	1200	1.5	A	1.93	0.25	31.42
5	1200	2.1	A	2.27	0.59	74.14
6	1200	2.7	A	2.41	0.73	91.73
7	1400	1.5	A	1.99	0.21	30.79
8	1400	2.1	A	2.51	0.73	107.02
9	1400	2.7	A	2.95	1.17	171.53
10	1000	1.5	B	1.70	0.13	13.61
11	1000	2.1	B	2.17	0.60	62.83
12	1000	2.7	B	2.29	0.72	75.40
13	1200	1.5	B	1.85	0.17	21.36
14	1200	2.1	B	2.21	0.53	66.60
15	1200	2.7	B	2.46	0.78	98.02
16	1400	1.5	B	1.99	0.21	30.79
17	1400	2.1	B	2.53	0.75	109.96
18	1400	2.7	B	2.66	0.88	129.01

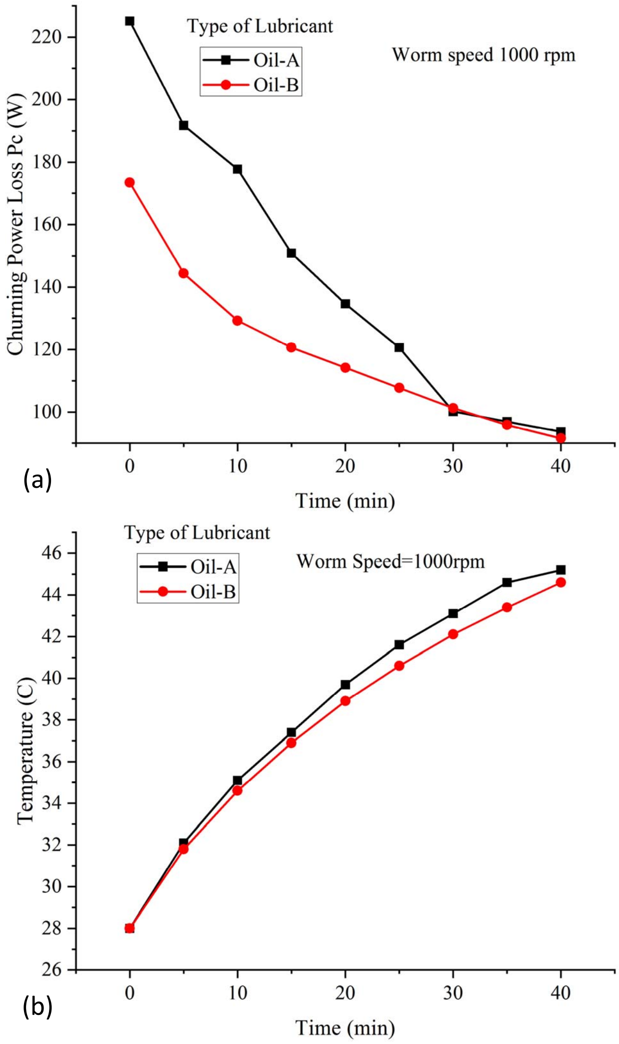

Figure 7. Influence of lubricant type on churning power loss and heating rate when the worm is at the bottom position, worm speed is 1000 rpm and lubricant volume is 2.7 litres: (a) churning power loss vs. time and (b) lubricant temperature vs. time

Table 5. Heating rate and churning power loss for different lubricants, worm speed of 1000 rpm, lubricant volume of 2.7 litres and worm at the bottom position

Time, minutes	Oil A		Oil B	
	Tempe-rature, °C	Churning power loss, W	Tempe-rature, °C	Churning power loss, W
0	28.0	225.12	28.0	173.42
5	32.1	191.73	31.8	144.33
10	35.1	177.72	34.6	129.25
15	37.4	150.80	36.9	120.64
20	39.7	134.64	38.9	114.17
25	41.6	120.64	40.6	107.71
30	43.1	100.17	42.1	101.25
35	44.6	96.94	43.4	95.86
40	45.2	93.71	44.6	91.55

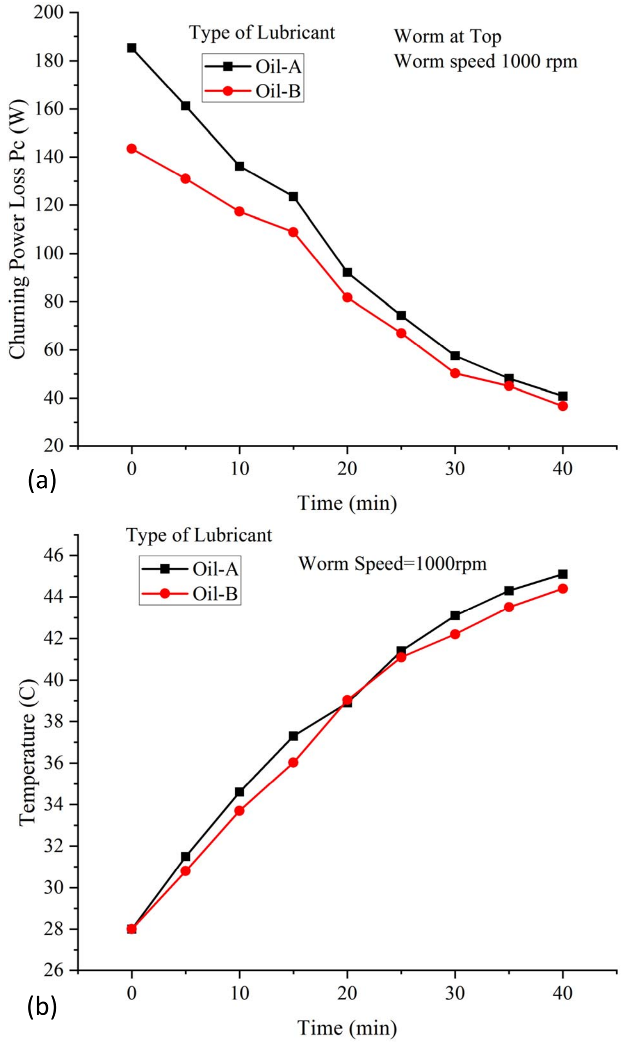

Figure 8. Influence of lubricant type on churning power loss and heating rate when the worm is at the top position, worm speed is 1000 rpm and lubricant volume is 2.7 litres: (a) churning power loss vs. time and (b) lubricant temperature vs. time

Table 6. Heating rate and churning power loss for different lubricants, worm speed of 1000 rpm, lubricant volume of 2.7 litres and worm at the top position

Time, minutes	Oil A		Oil B	
	Tempe-rature, °C	Churning power loss, W	Tempe-rature, °C	Churning power loss, W
0	28.0	185.35	28.0	143.47
5	31.5	161.27	30.8	130.90
10	34.6	136.14	33.7	117.29
15	37.3	123.57	36.0	108.91
20	38.9	92.15	39.0	81.68
25	41.4	74.35	41.1	67.02
30	43.1	57.60	42.2	50.27
35	44.3	48.17	43.5	45.03
40	45.1	40.84	44.4	36.65

4. Conclusions

For worm gear, it was previously believed that the churning power losses had insufficient effects and could be disregarded. An extensive experimental investigation is carried out using mineral and synthetic oil to explore the churning power loss for dip-lubricated worm gear in order to test this idea. The main focus was to calculate the churning power loss and highlight the effect of lubricant type (mineral oil and synthetic oil) on churning power loss. The direct torque measuring technique was used with the help of a developed test rig. The trials unequivocally demonstrated that the worm gear has churning power loss, which has a significant impact on efficiency. The impact of the lubricant type was also analysed.

Mineral and synthetic lubricants with nearly equal viscosities have not had a significant impact on the churning power loss of worm gears. With time, the rates of heating for both oils are nearly equal. When compared to synthetic oil under the same operating conditions, mineral oil initially produced 20 – 25 % more churning power loss. This occurs because the viscosity is less sensitive to variations in pressure and temperature. Worm gears operate in the boundary and mixed lubrication regime. Thus, the effect of the oil lubricity is also quite important. Synthetic oils have a much better lubricity than mineral oils, which initially results in considerably better performance. However, this difference decreased to 2 – 5 % as the temperature rose. It proves that churning power loss is not primarily caused by the type of lubricant with similar viscosity used at the same temperature.

Reducing the load-independent losses by taking windage losses, oil seal losses and bearing churning losses into consideration may be the main goal of future studies in this area. It is advised to look into the churning power loss for worm gear with jet lubrication. There is a great deal of possibilities for a more thorough numerical study, such as many that could theoretically be achieved by CFD.

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